CPEN 400D: Deep Learning

Lecture 9: Generative Adversarial Networks

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University of British Columbia

Winter, Term 2, 2022

Outline

- **Motivation**
- GANs
  - Overview
  - Minimax Loss
  - Properties
  - Architectures
  - Challenges of GANs
- GANs Variants
  - Wasserstein GANs
  - Progressive GANs
  - Cycle GANs

Deep Generative Models

Deep generative models are the most exciting area in deep/machine learning, AI…

Models can even generate the reflection in the puddle!

Image Credit: Midjourney V5

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We have learned deep generative models like deep auto-regressive models (e.g., GPT series) and variational auto-encoders (VAEs).

They are trained to maximize the log likelihood of observed data (e.g., next tokens) or its lower bound (ELBO).

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No, we have adversarial learning, score matching / denoising diffusion, moment matching, and so on.

The beauty of deep generative models is *all models are wrong, but many are useful!*

Image Credit: Midjourney V5

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Overview

Generative Adversarial Networks (GANs) [1]

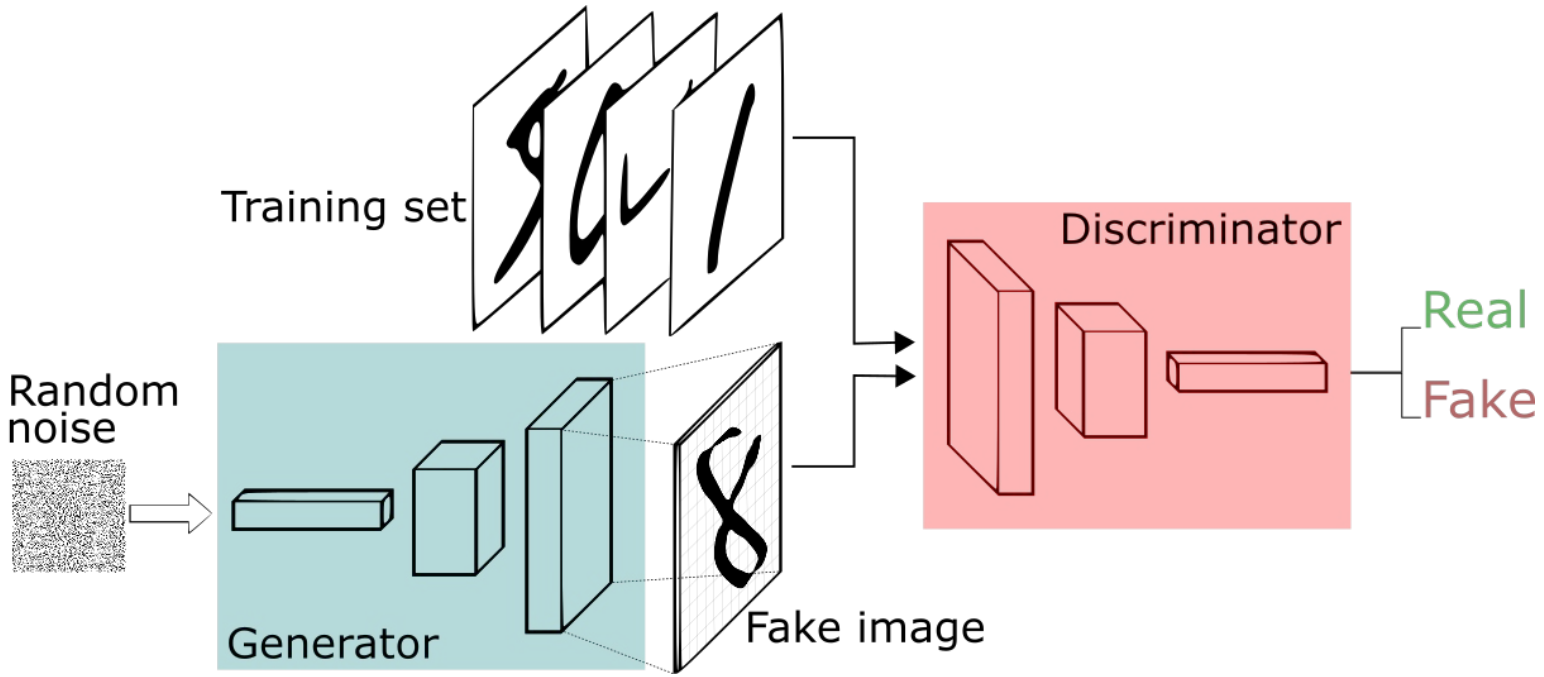

Image Credit: [2]

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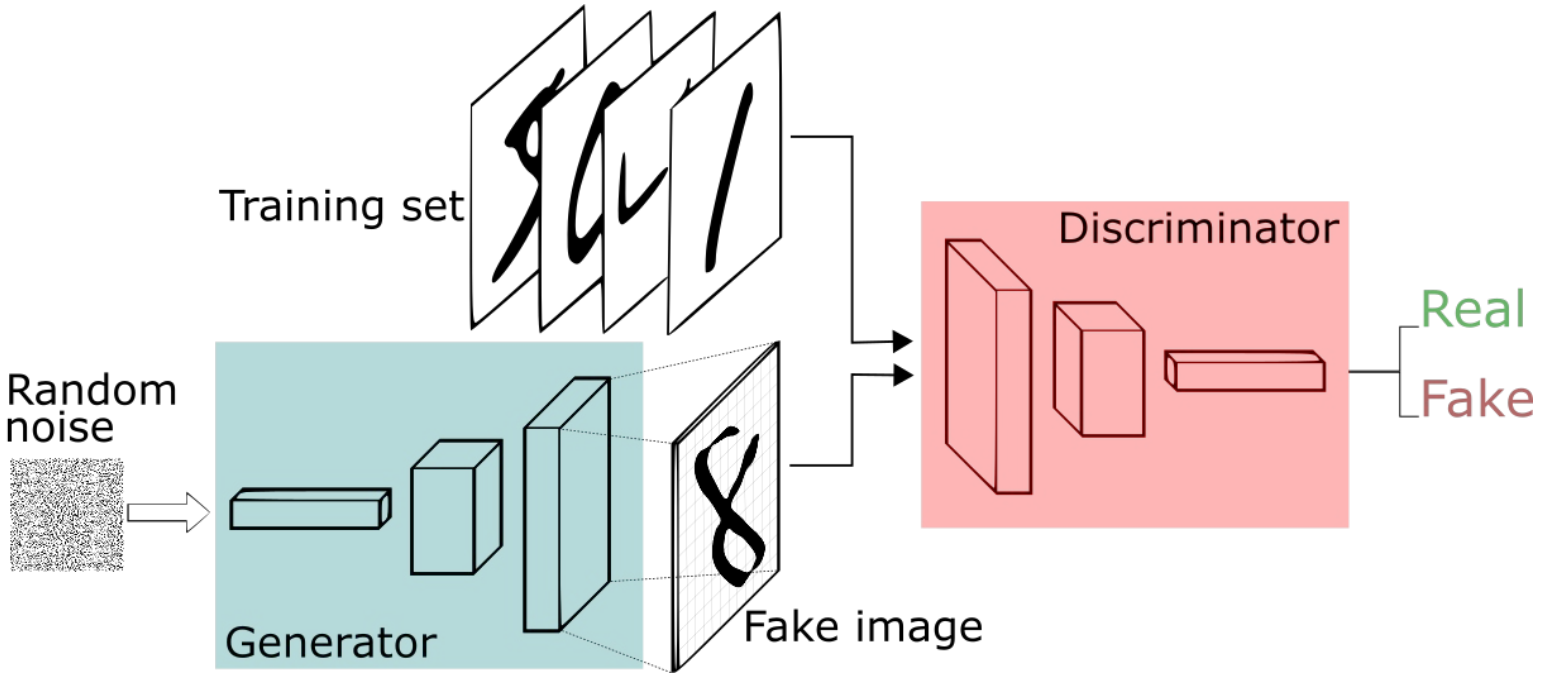
Minimax Loss

GANs [1] : Two neural networks (generator and discriminator) contest with each other in the form of a zero-sum game, where one agent's gain is another agent's loss.



Learning: $\min_{\theta} \max_{\phi} \quad \mathbb{E}_{X \sim p_{\text{data}}(X)}[\log D_{\phi}(X)] + \mathbb{E}_{Z \sim p(Z)}[\log(1 - D_{\phi}(G_{\theta}(Z))]$

Image Credit: [2]

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Properties of GANs

Generative Adversarial Networks (GANs) [1]

1. Fix generator, the optimal discriminator is $$D^*_\phi(X) = \frac{p_{\text{data}}(X)}{p_{\text{data}}(X) + p_{G_\theta}(X)}$$

Properties of GANs

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Why?

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Why?

$$\begin{aligned}
\ell(G_\theta, D_\phi) &= \mathbb{E}_{X \sim p_{\text{data}}(X)}[\log D_\phi(X)] + \mathbb{E}_{Z \sim p(Z)}[\log(1 - D_\phi(G_\theta(Z))] \\
&= \mathbb{E}_{X \sim p_{\text{data}}(X)}[\log D_\phi(X)] + \mathbb{E}_{X \sim p_{G_\theta}(X)}[\log(1 - D_\phi(X))] \\
&= \int p_{\text{data}}(X) \log D_\phi(X) + p_{G_\theta}(X) \log(1 - D_\phi(X)) dX
\end{aligned}$$

Properties of GANs

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\end{aligned}$$

Law Of The Unconscious Statistician (LOTUS)

Set the gradient of loss w.r.t. D to be zero

Properties of GANs

Generative Adversarial Networks (GANs) [1]

$$
\begin{aligned}
C(G_\theta) &= \max_{D_\phi} \quad \ell(G_\theta, D_\phi) \\
&= \mathbb{E}_{X \sim p_{\text{data}}(X)} \left[\log D^*_\phi(X)\right] + \mathbb{E}_{X \sim p_{G_\theta}(X)} \left[\log(1 - D^*_\phi(X))\right] \\
&= \mathbb{E}_{X \sim p_{\text{data}}(X)} \left[\log \left(\frac{p_{\text{data}}(X)}{p_{\text{data}}(X) + p_{G_\theta}(X)}\right)\right] + \mathbb{E}_{X \sim p_{G_\theta}(X)} \left[\log \left(\frac{p_{G_\theta}(X)}{p_{\text{data}}(X) + p_{G_\theta}(X)}\right)\right]
\end{aligned}
$$

Properties of GANs

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\end{aligned}
$$

2. The global minimum of $C(G_\theta)$ is achieved iff. $p_{\text{data}}(X) = p_{G_\theta}(X)$

Why?

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$$

2. The global minimum of $C(G_\theta)$ is achieved iff. $p_{\text{data}}(X) = p_{G_\theta}(X)$

Why?

$$
\begin{aligned}
C(G_\theta) = &\mathbb{E}_{X \sim p_{\text{data}}(X)} \left[\log \left(\frac{p_{\text{data}}(X)}{(p_{\text{data}}(X) + p_{G_\theta}(X))/2}\right)\right] \\
&+ \mathbb{E}_{X \sim p_{G_\theta}(X)} \left[\log \left(\frac{p_{G_\theta}(X)}{(p_{\text{data}}(X) + p_{G_\theta}(X))/2}\right)\right] + 2\log(\frac{1}{2}) \\
&= \text{JSD}(p_{\text{data}}(X) \| p_{G_\theta}(X)) - \log(4)
\end{aligned}
$$

Properties of GANs

Generative Adversarial Networks (GANs) [1]

2. The global minimum of $C(G_\theta)$ is achieved iff. $p_{\text{data}}(X) = p_{G_\theta}(X)$

Why?

$$
\begin{aligned}
C(G_\theta) &= \mathbb{E}_{X \sim p_{\text{data}}(X)} \left[\log \left(\frac{p_{\text{data}}(X)}{(p_{\text{data}}(X) + p_{G_\theta}(X))/2} \right) \right] \\
&\quad + \mathbb{E}_{X \sim p_{G_\theta}(X)} \left[\log \left(\frac{p_{G_\theta}(X)}{(p_{\text{data}}(X) + p_{G_\theta}(X))/2} \right) \right] + 2 \log(\frac{1}{2}) \\
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\end{aligned}
$$

Jensen–Shannon divergence is non-negative and is zero iff. P = Q

$$
\text{JSD}(P\|Q) = \frac{1}{2}\text{KL}(P\|\frac{P+Q}{2}) + \frac{1}{2}\text{KL}(Q\|\frac{P+Q}{2})
$$

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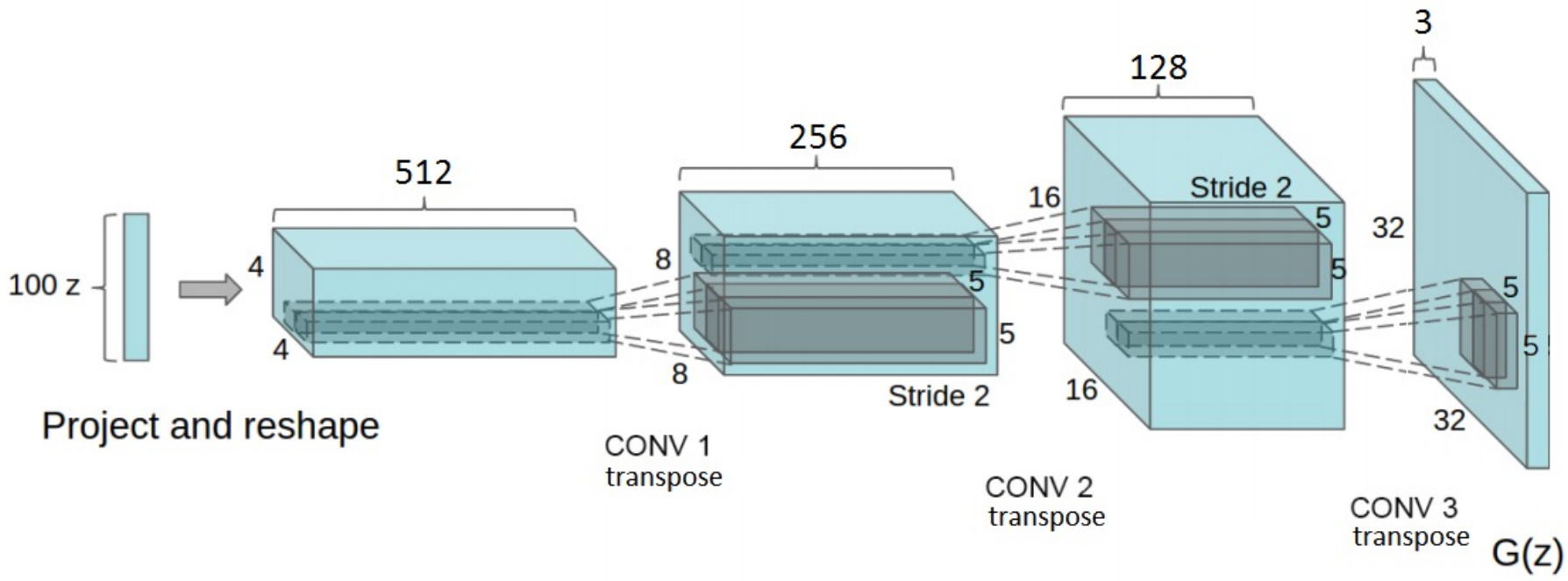
Architecture

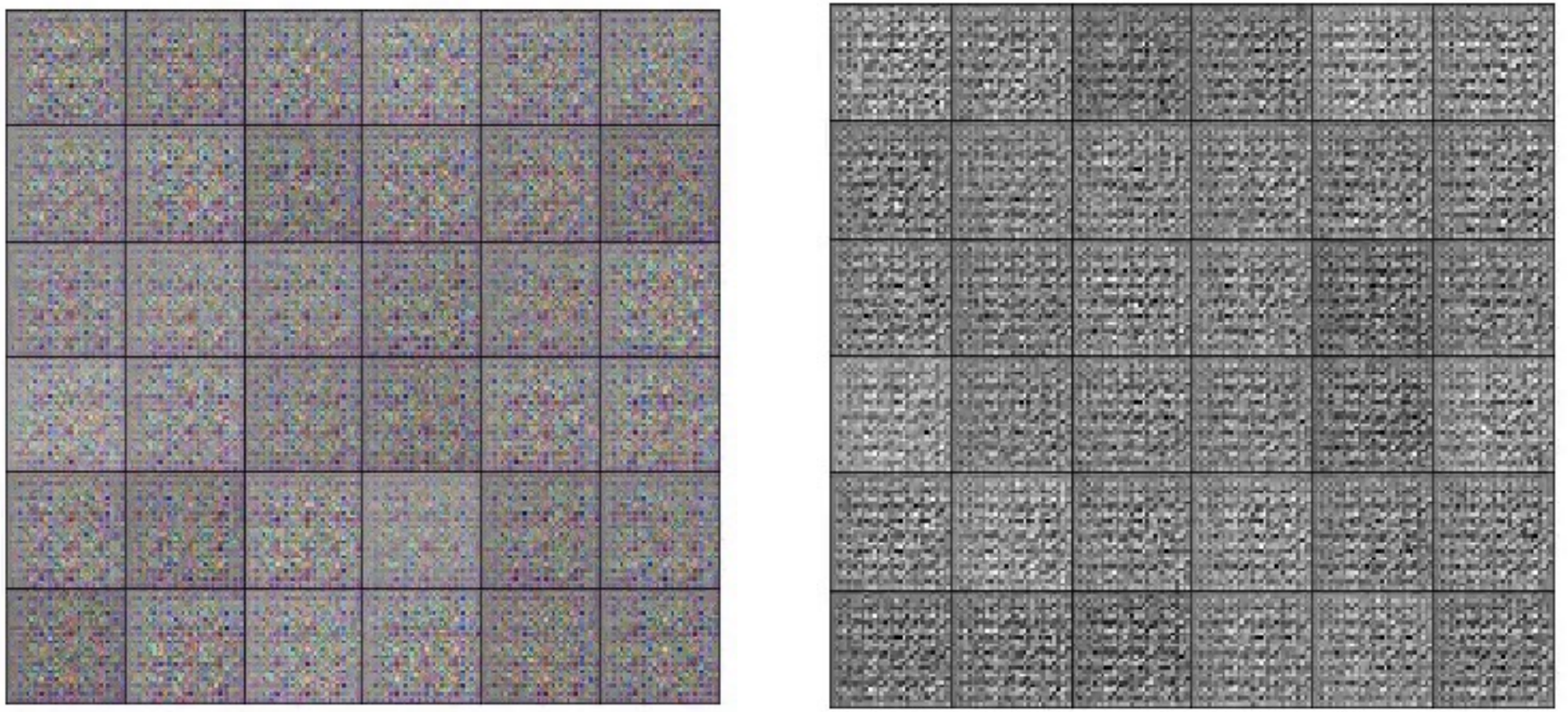
Deep Convolutional Generative Adversarial Network (DCGANs) [3] : using CNNs as both Generator and Discriminator.



Generator

Image Credit: [2]

Demo of GANs



Samples from generator during training on SVHNs (left) and MNIST (right)

Image Credit: [2]

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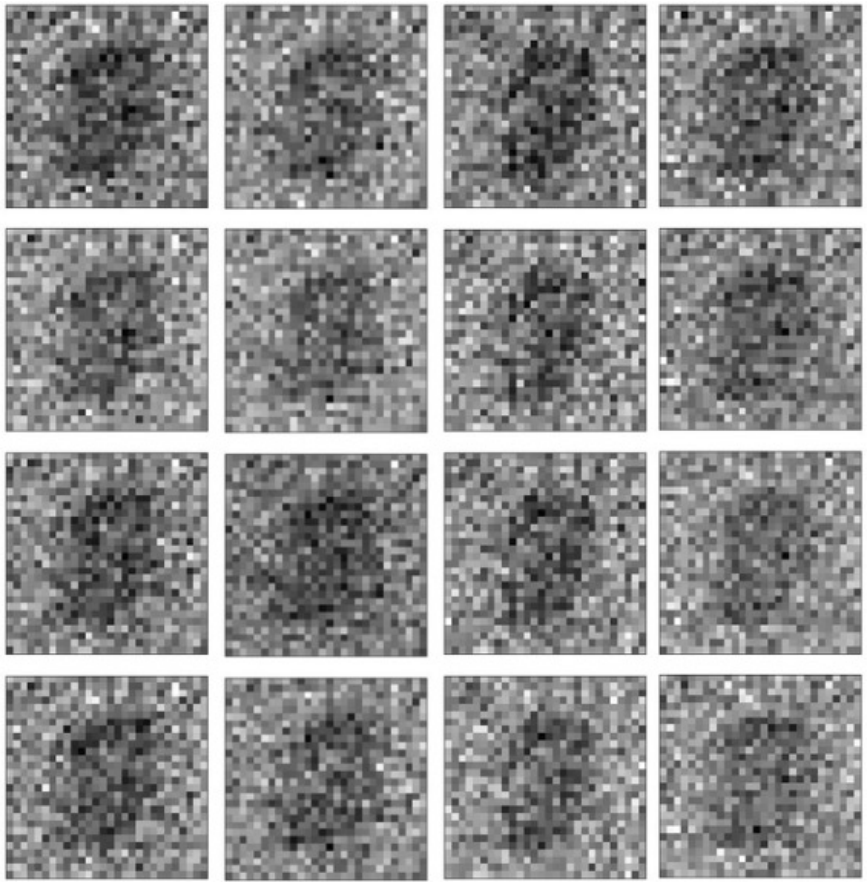
Challenges in Training GANs

Two common problems in training GANs are: *training instability*



Convergence Failure: e.g., caused by imbalance training of generator and discriminator

Image Credit: [4]

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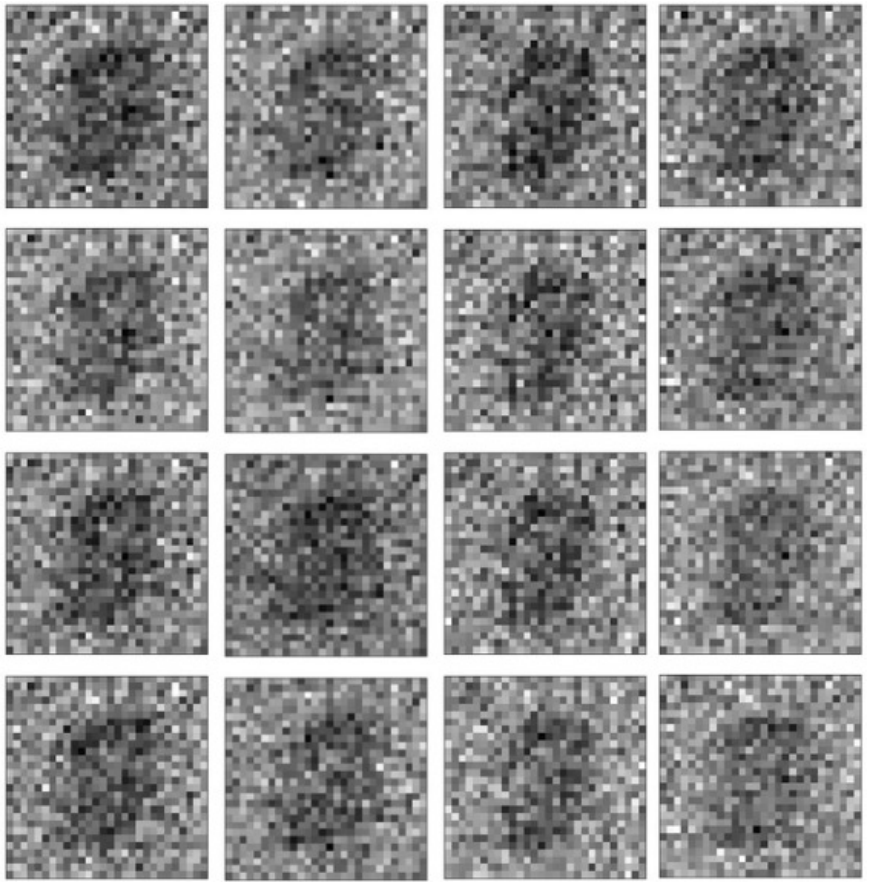
Two common problems in training GANs are: *training instability* and *mode collapse*



Convergence Failure: e.g., caused by imbalance training of generator and discriminator

Mode Collapse: generating samples that are very similar or even identical

Image Credit: [4]

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Wasserstein GANs

Earth Mover Distance / Wasserstein Metric:

$$\text{EMD}(P_r, P_\theta) = \inf_{\gamma \in \Pi} \sum_{x,y} \|x - y\| \gamma(x, y) = \inf_{\gamma \in \Pi} \mathbb{E}_{(x,y) \sim \gamma} \|x - y\|$$

$$= \inf_{\gamma \in \Pi} \langle \mathbf{D}, \mathbf{\Gamma} \rangle_{\text{F}}$$

$\prod$ is the set of all distributions whose marginals are P_r, P_θ respectively.

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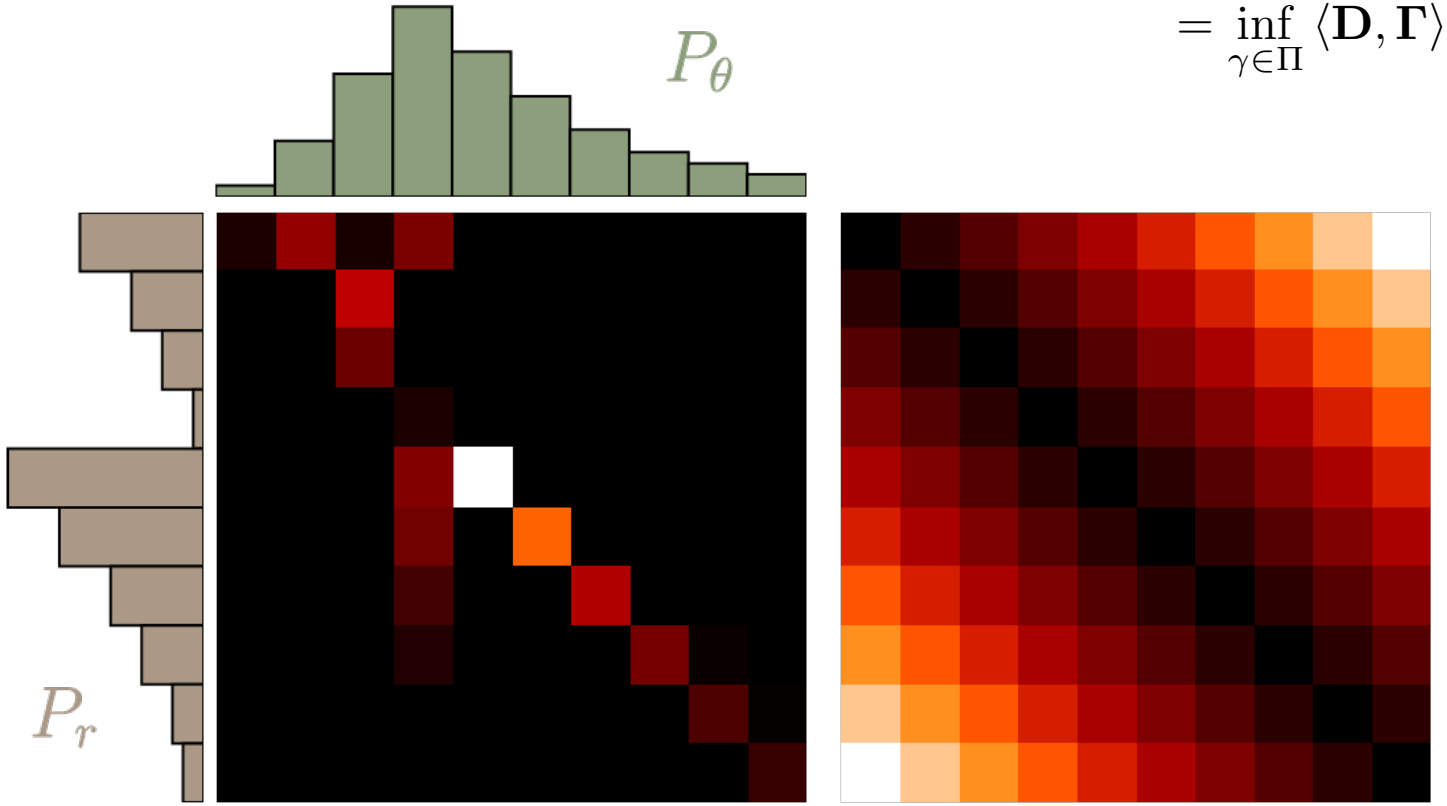

$\mathbf{\Gamma}$

$\mathbf{\Gamma} = \gamma(x, y)$

$\mathbf{D}$

$\mathbf{D} = \|x - y\|$

Image Credit: [5]

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Wasserstein distance (using Kantorovich-Rubinstein duality):

$$\text{EMD}(P_r, P_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x\sim P_r} f(x) - \mathbb{E}_{x\sim P_\theta} f(x).$$

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Wasserstein-GAN [6] proposes a unified objective:

Learn Discriminator via $$\max_{\phi} \quad \mathbb{E}_{X\sim p_{\text{data}}(X)}[D_\phi(X)] - \mathbb{E}_{\epsilon\sim p(\epsilon)}[D_\phi(G_\theta(\epsilon))]$$

Learn Generator via $$\min_{\theta} \quad \mathbb{E}_{X\sim p_{\text{data}}(X)}[D_\phi(X)] - \mathbb{E}_{\epsilon\sim p(\epsilon)}[D_\phi(G_\theta(\epsilon))]$$

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To enforce Lipschitz condition, one can clip weights [6], add gradient penalty (WGAN-GP) [7], and use spectral normalization [8]

Wasserstein GANs

Image Credit: [7]

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Progressive GANs

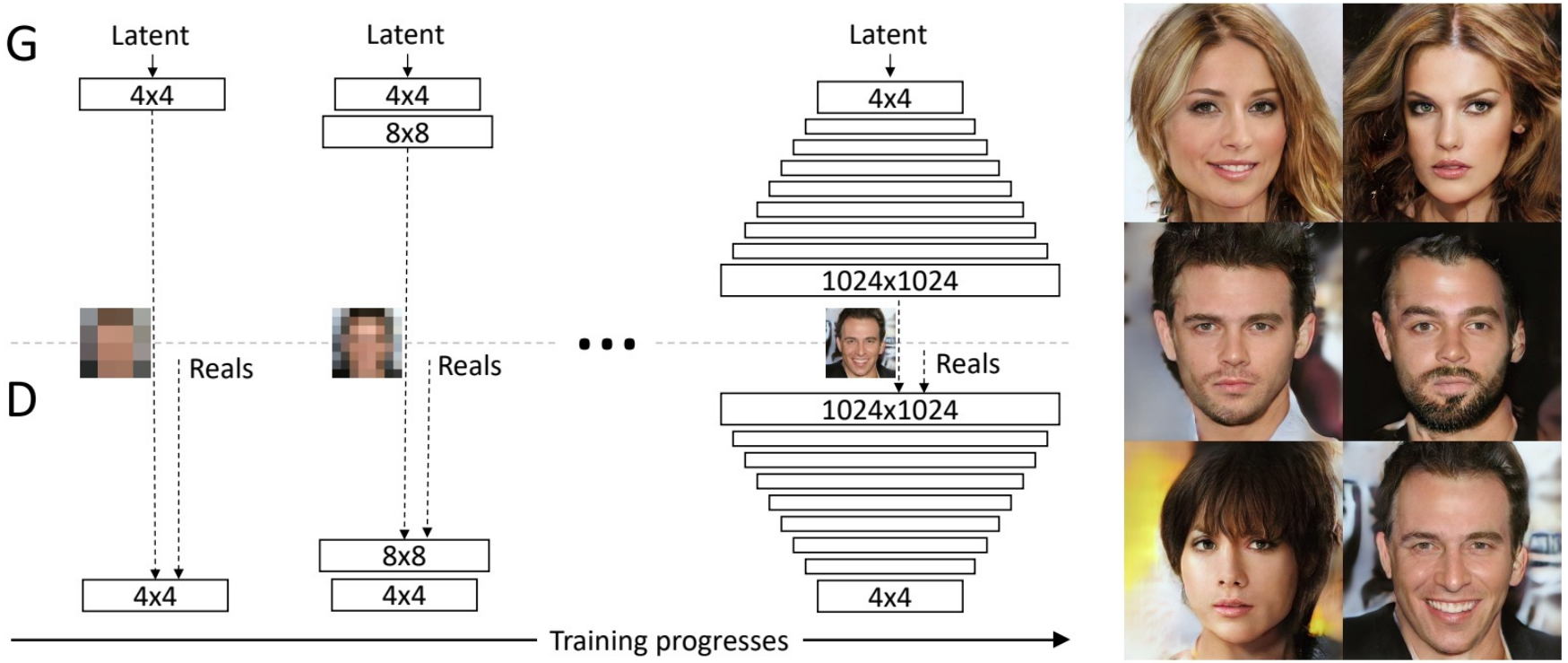

Figure 1: Our training starts with both the generator (G) and discriminator (D) having a low spatial resolution of 4×4 pixels. As the training advances, we incrementally add layers to G and D, thus increasing the spatial resolution of the generated images. All existing layers remain trainable throughout the process. Here $N \times N$ refers to convolutional layers operating on $N \times N$ spatial resolution. This allows stable synthesis in high resolutions and also speeds up training considerably. One the right we show six example images generated using progressive growing at 1024×1024.

Image Credit: [9]

Progressive GANs

Figure 5: 1024 × 1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

Image Credit: [9]

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Cycle GANs

Cycle-Consistent Generative Adversarial Networks [10] learn the image-to-image translation without a training set of aligned image pairs

Image Credit: [12]

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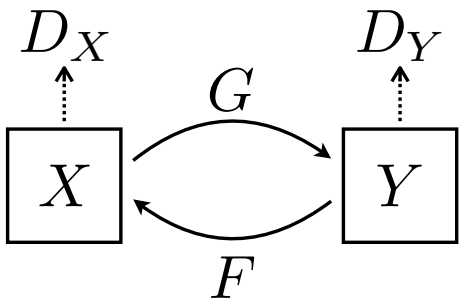

Image Credit: [10,12]

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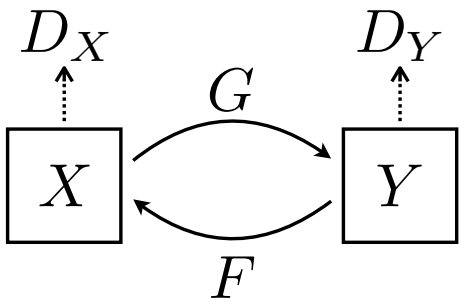

$$\mathcal{L}_{\text{GAN}}(F, D_X, X, Y) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\log D_X(x)] + \mathbb{E}_{y \sim p_{\text{data}}(y)}[\log(1 - D_X(F(y))]$$

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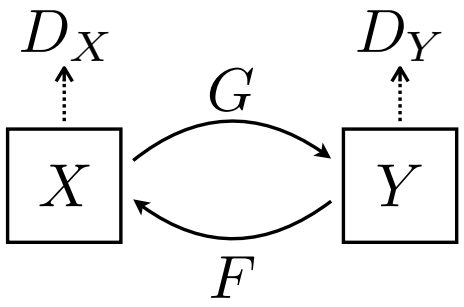

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Image Credit: [10,12]

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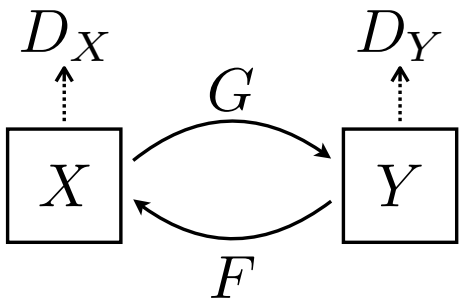

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$$\mathcal{L}_{\text{cyc}}(G, F) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\|F(G(x)) - x\|_1] + \mathbb{E}_{y \sim p_{\text{data}}(y)}[\|G(F(y)) - y\|_1]$$

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Cycle GANs

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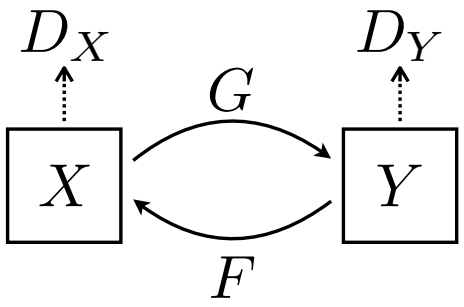

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$$\mathcal{L}(G, F, D_X, D_Y) = \mathcal{L}_{\text{GAN}}(G, D_Y, X, Y) + \mathcal{L}_{\text{GAN}}(F, D_X, Y, X) + \lambda \mathcal{L}_{\text{cyc}}(G, F)$$

Image Credit: [10,12]

Cycle GANs

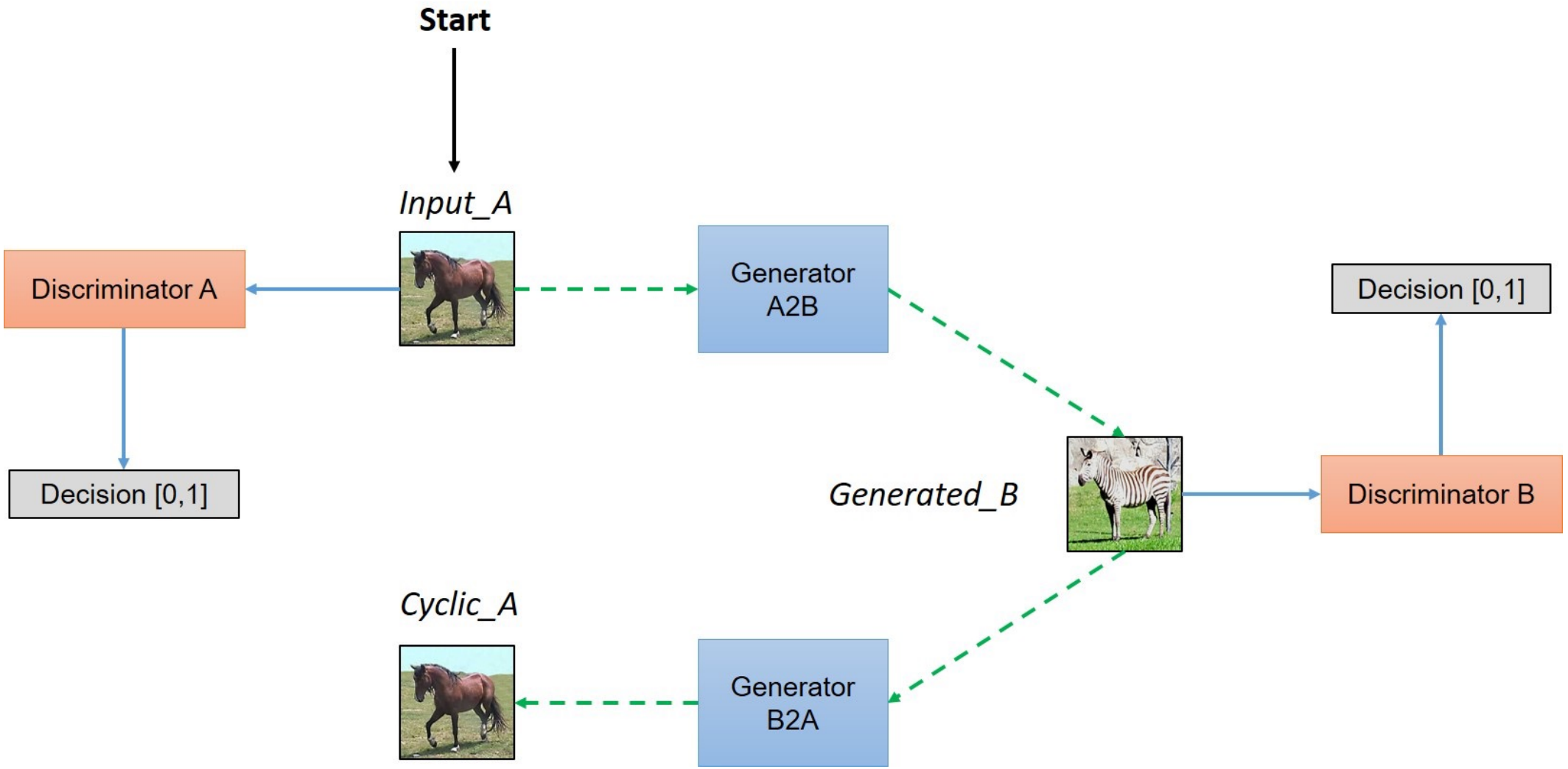

Image Credit: [9]

Cycle GANs

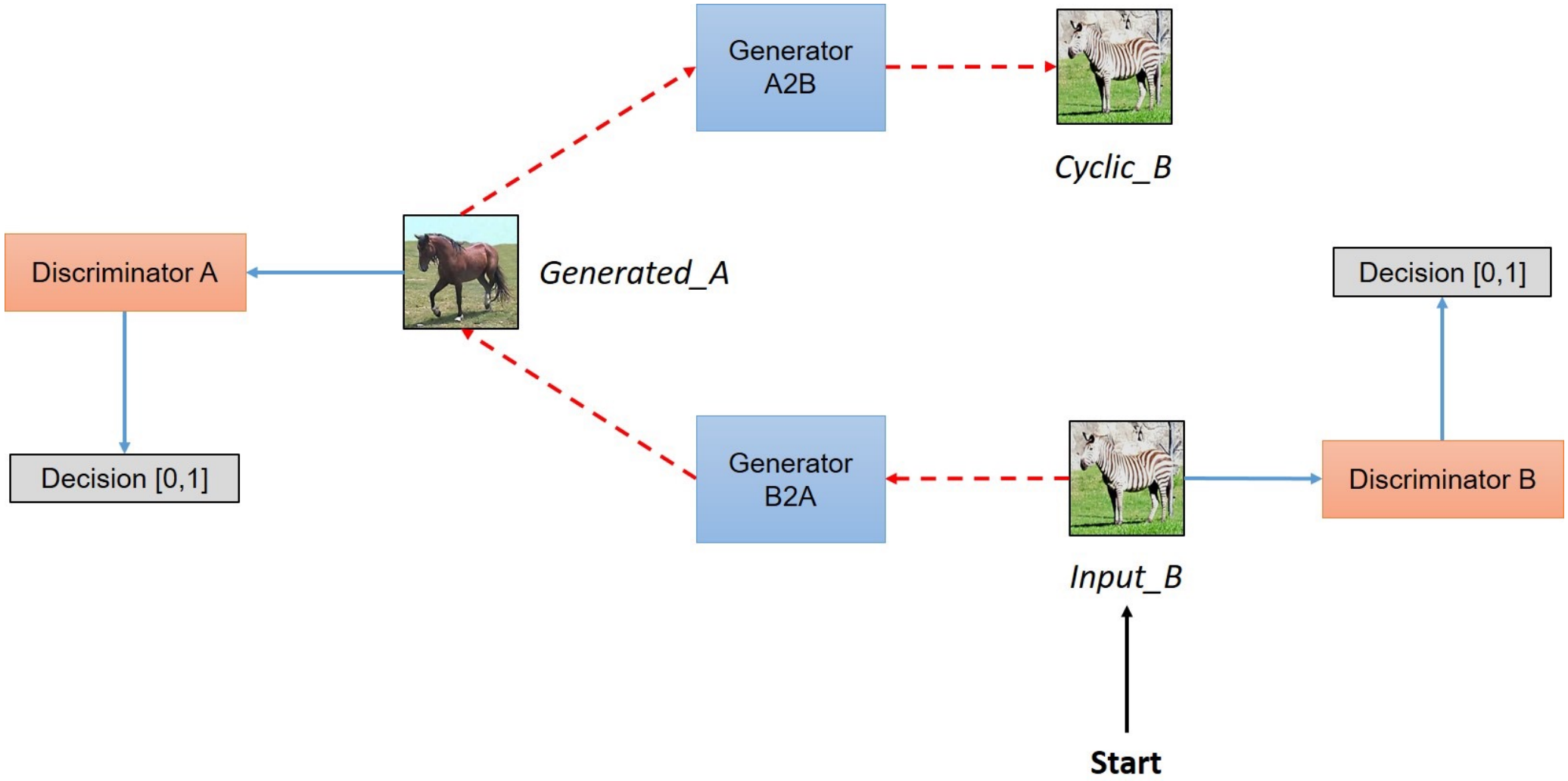

Image Credit: [9]

Cycle GANs

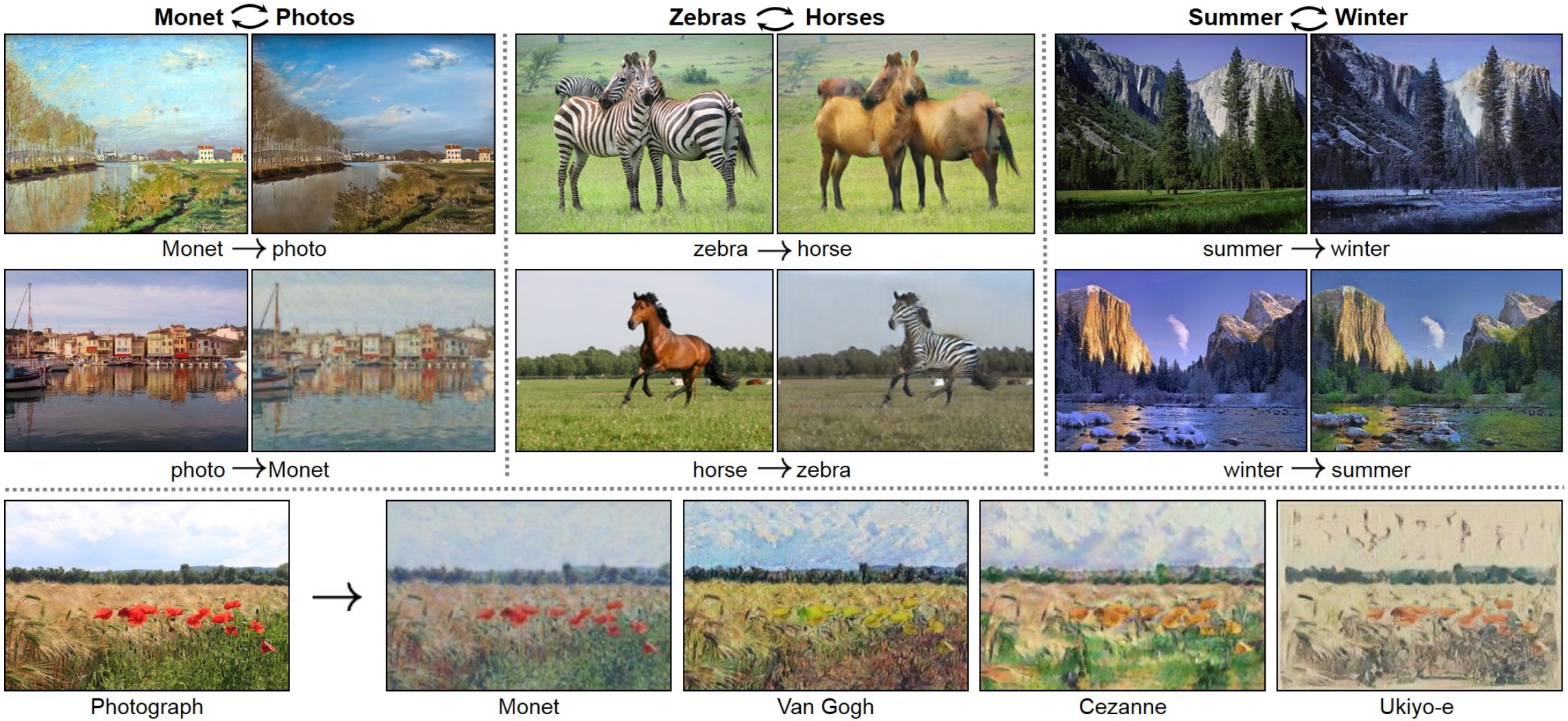

Image Credit: [12]

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Questions?