CPEN 400D: Deep Learning

Lecture 10: Deep Reinforcement Learning

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University of British Columbia

Winter, Term 2, 2022

Outline

- Reinforcement Learning
  - **Overview & Applications**
  - Key Concepts
  - Markov Decision Process (MDP) and its Extensions
  - Bellman Equation and its Optimality
  - RL Taxonomy
- Deep Reinforcement Learning
  - Q-learning & Deep Q-learning
  - Policy Gradient Methods

Reinforcement Learning (RL)

RL is about *learning to take actions that can maximize total future reward*!

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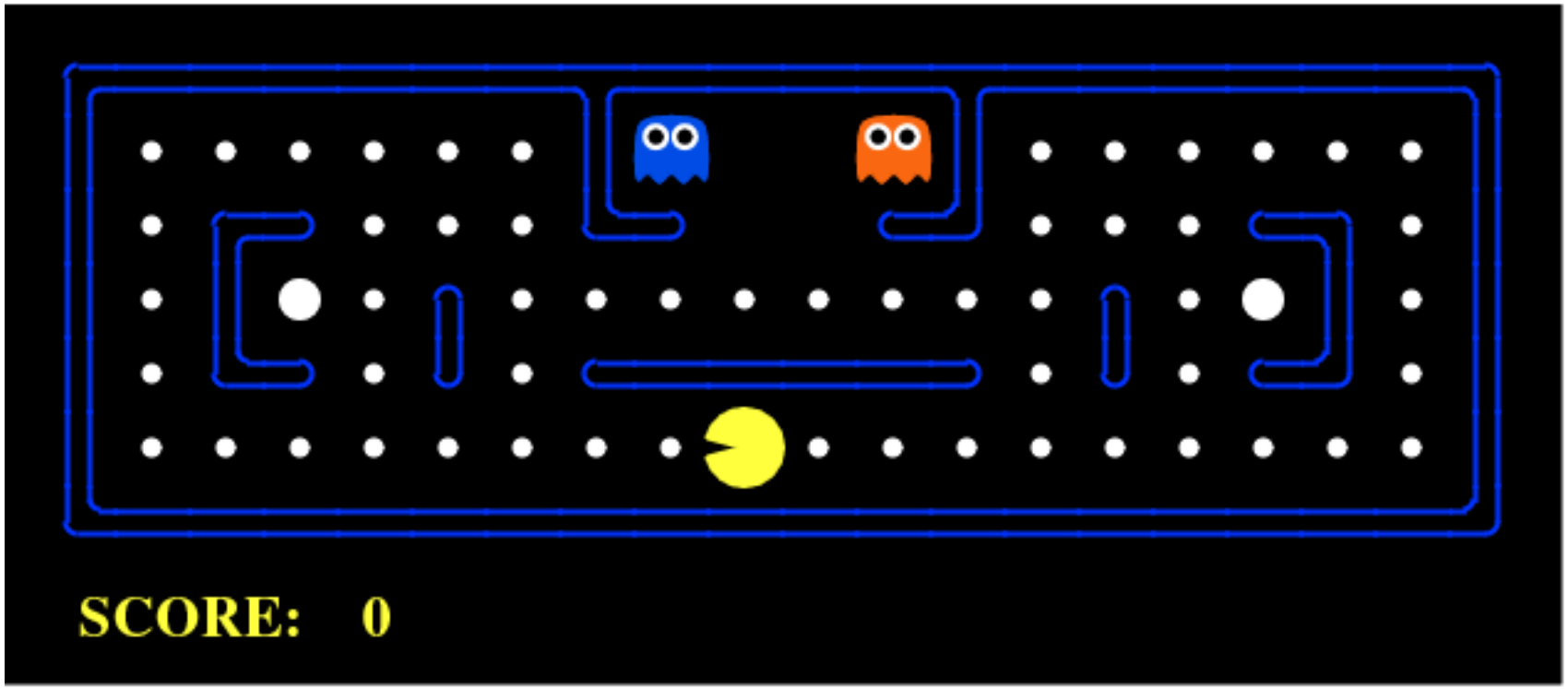

Image Credit: [1]

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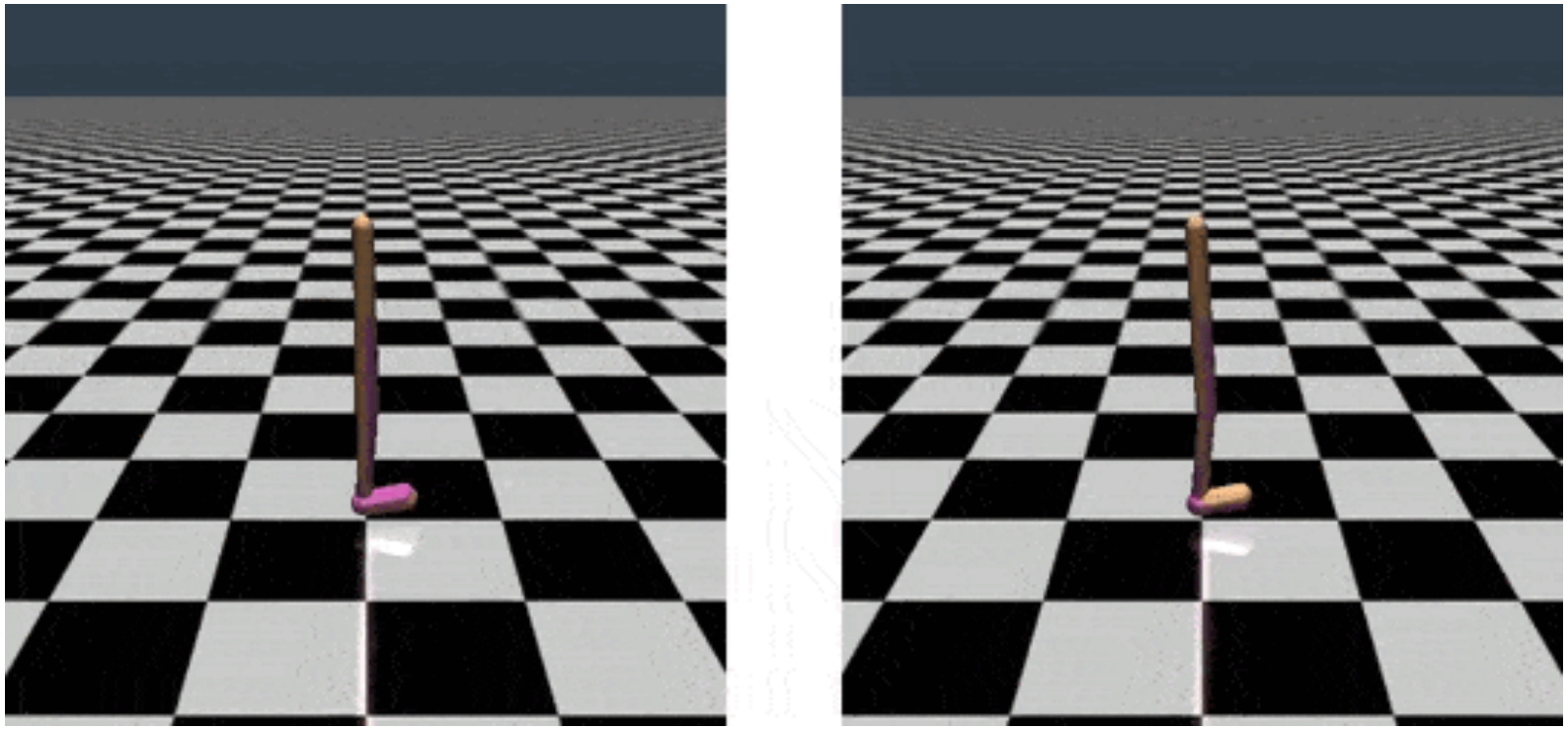

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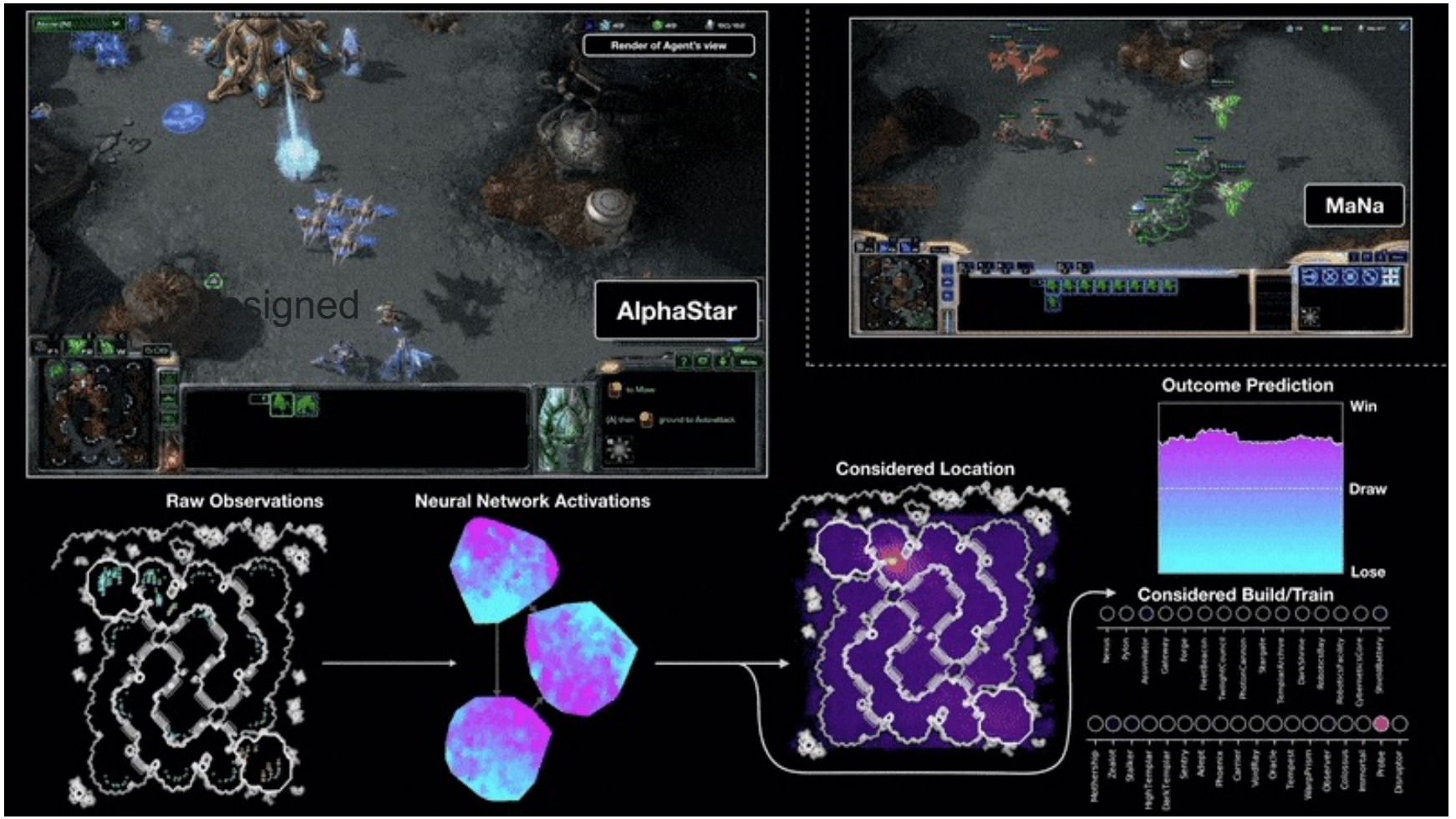

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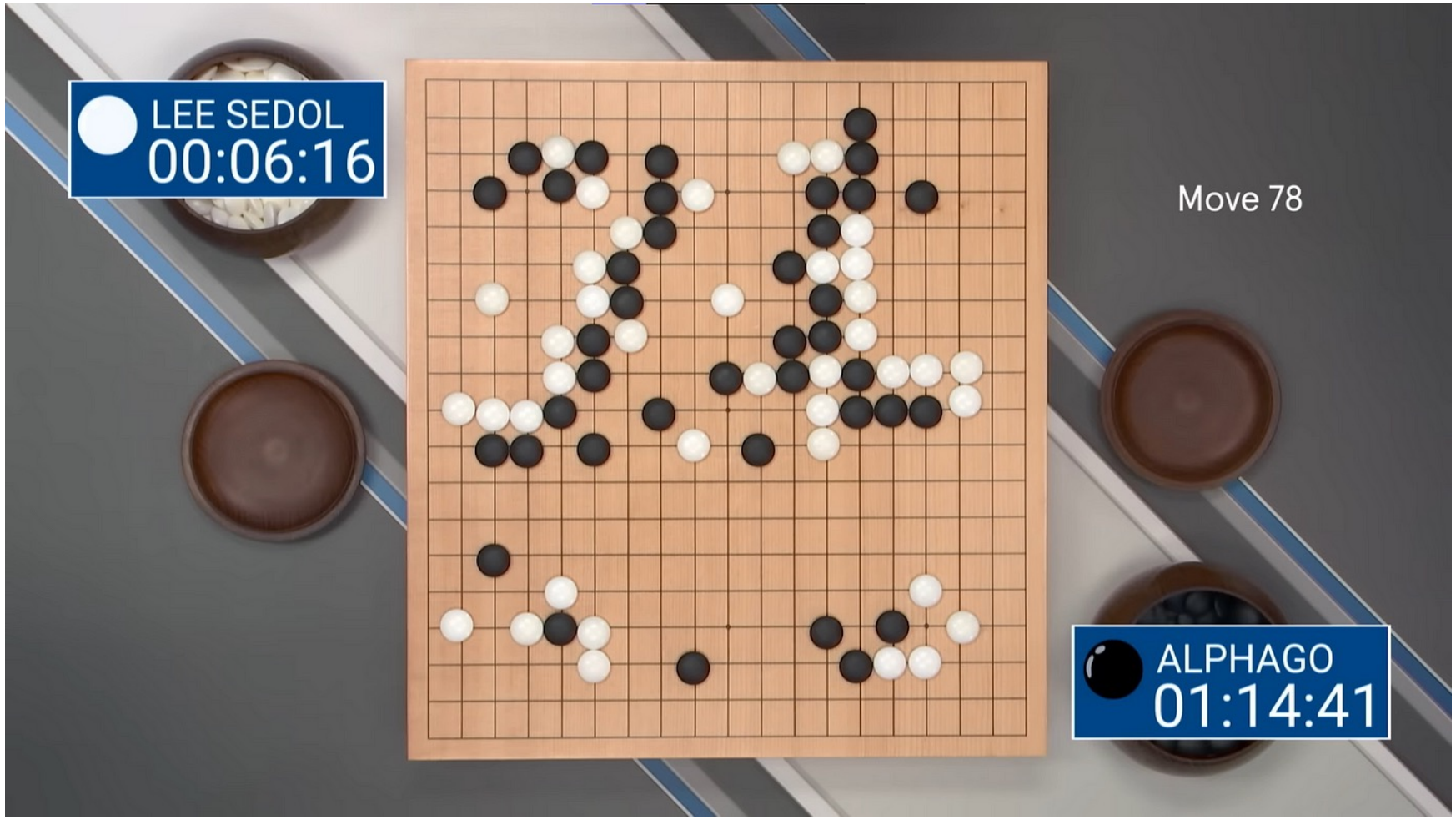

God's move: AlphaGo thought this move happens with 0.007% probability in human players!

This may be the last time a human go player beats AI!

Image Credit: [4]

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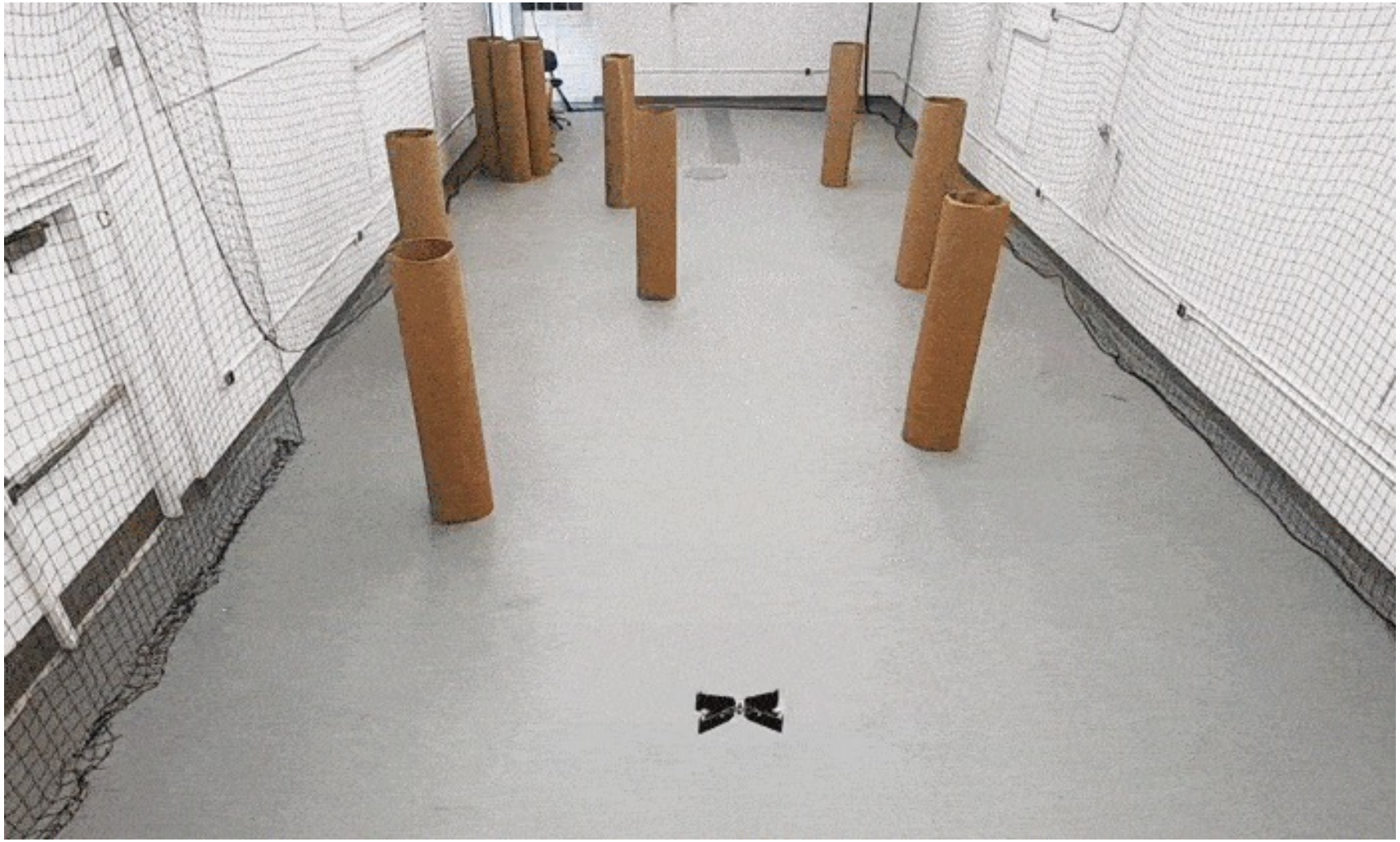

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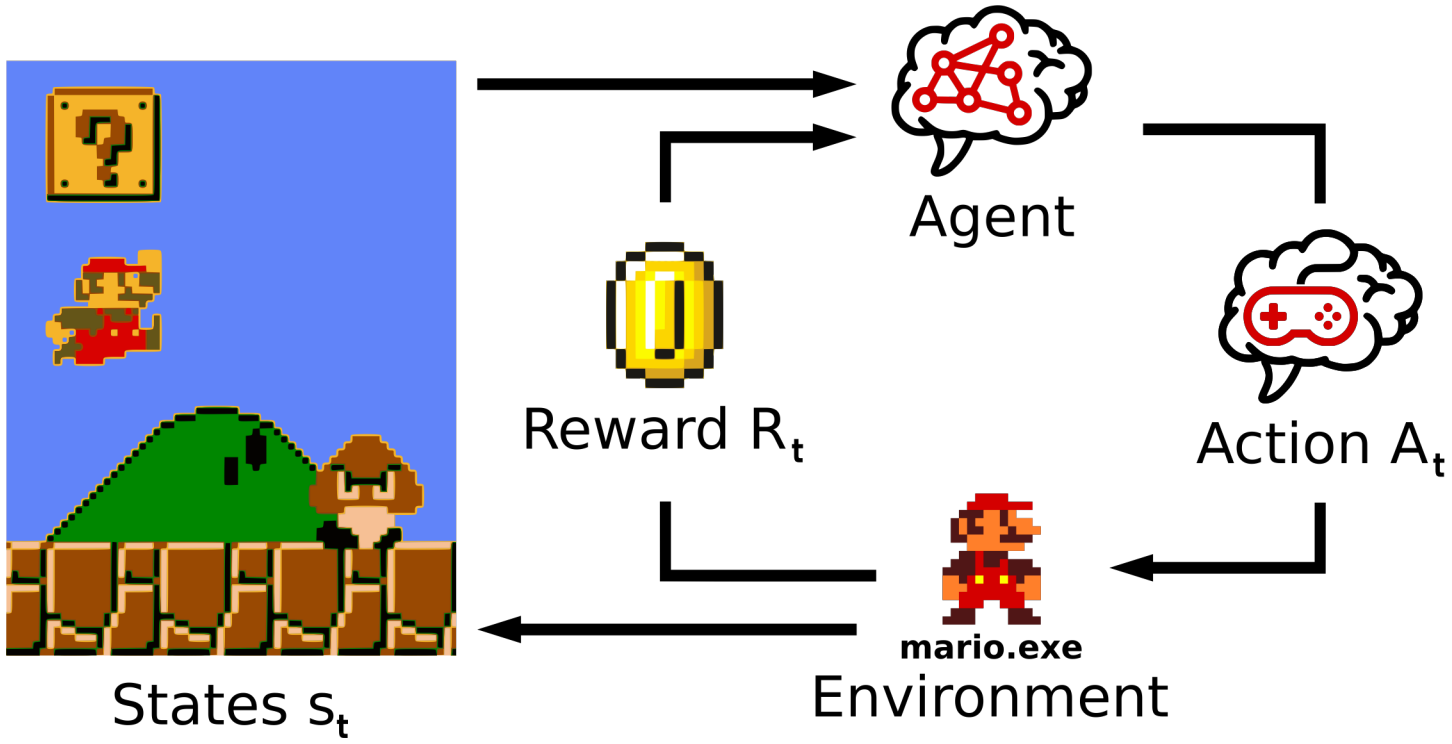
Let us look at the Super Mario example to grab the key concepts:

Agent: an intelligent program or a real robot



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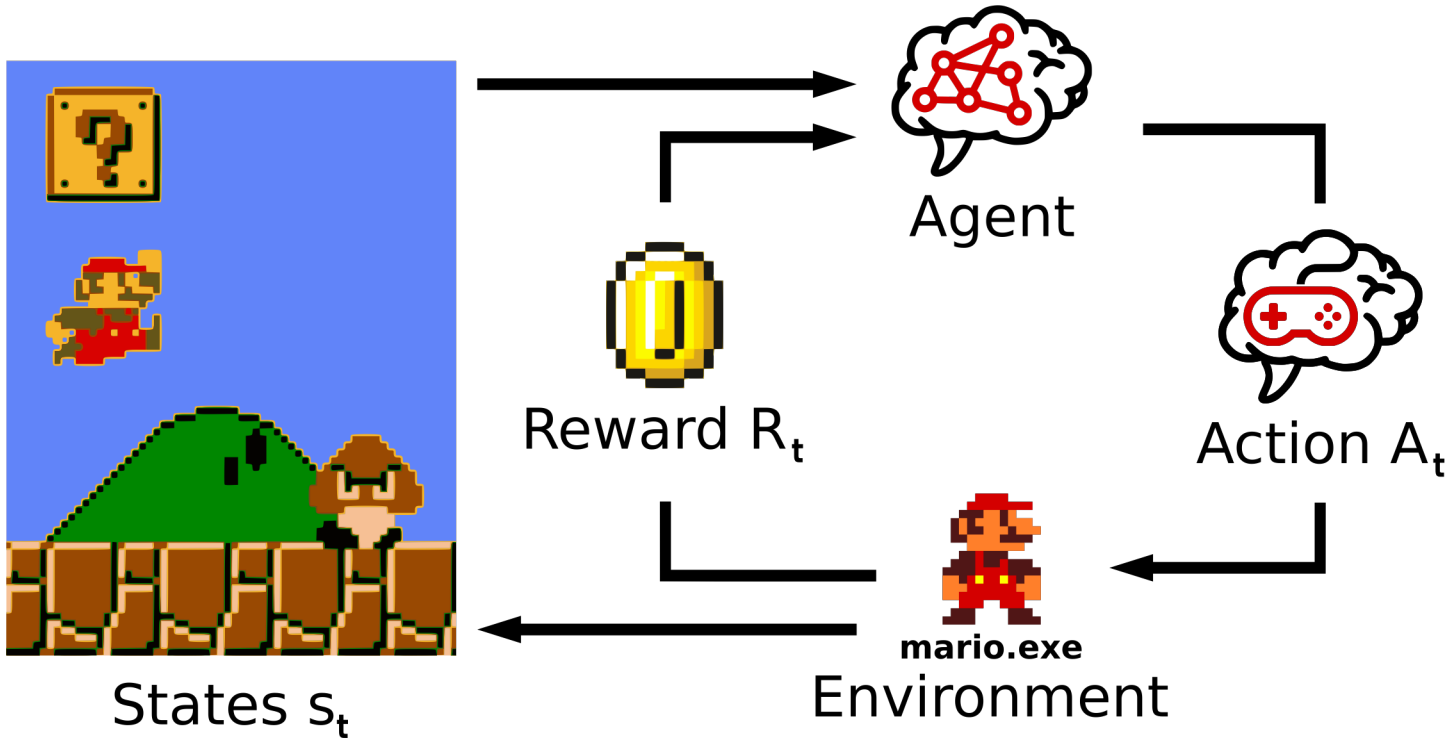

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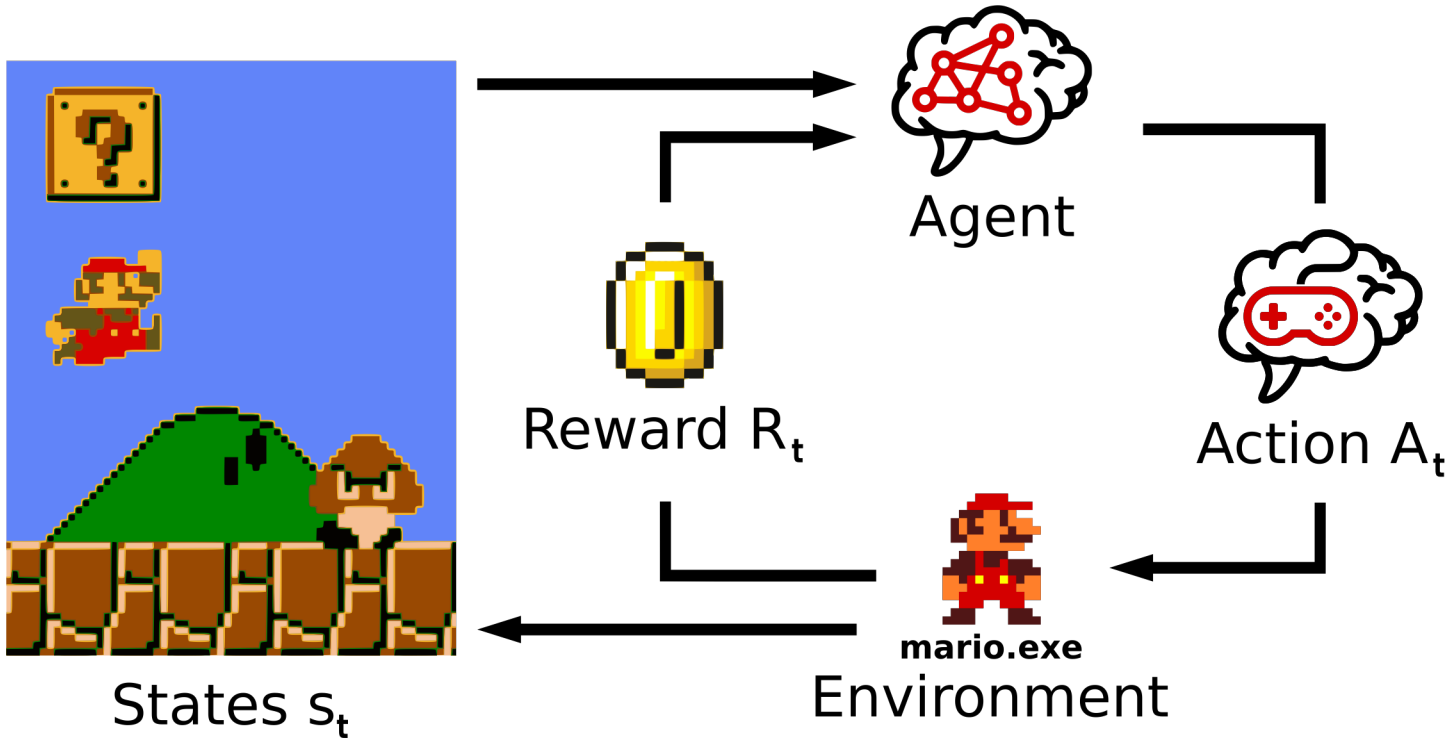

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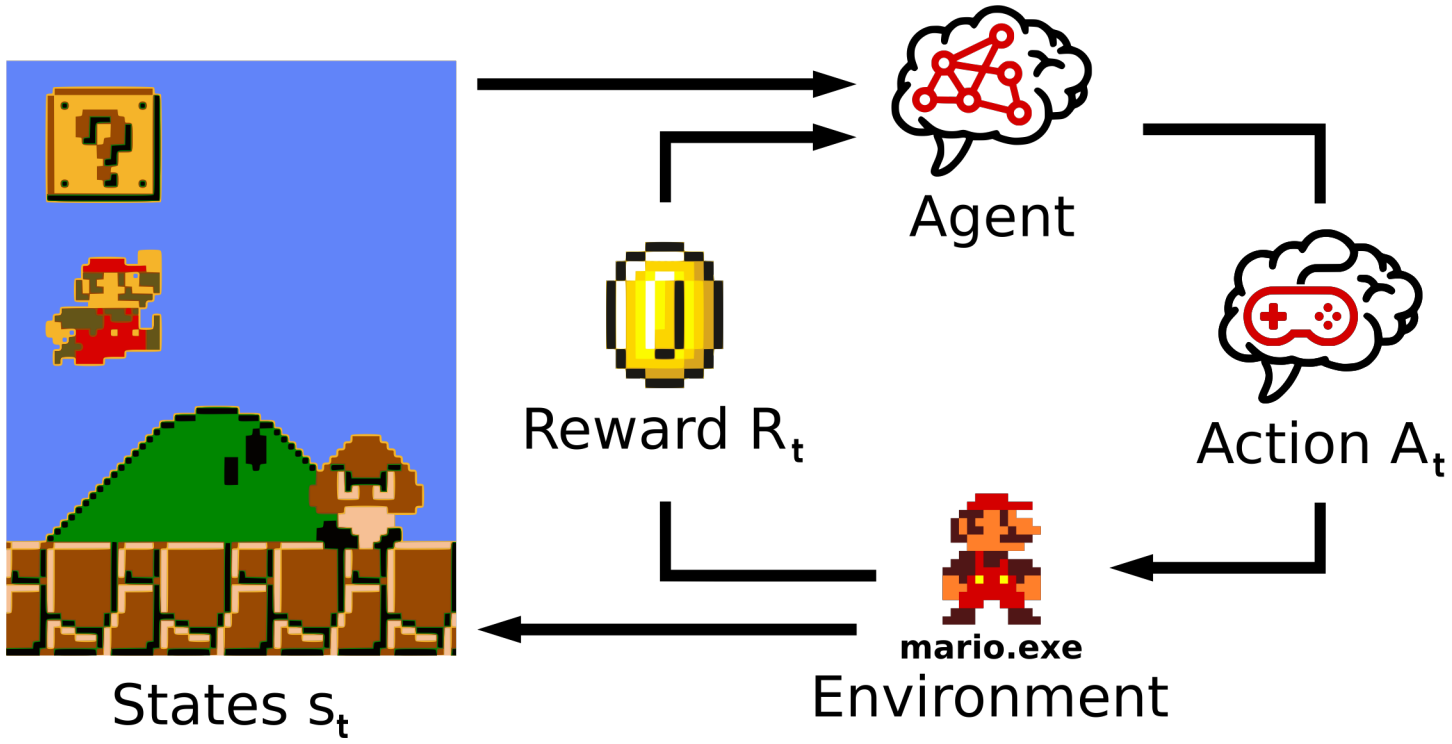

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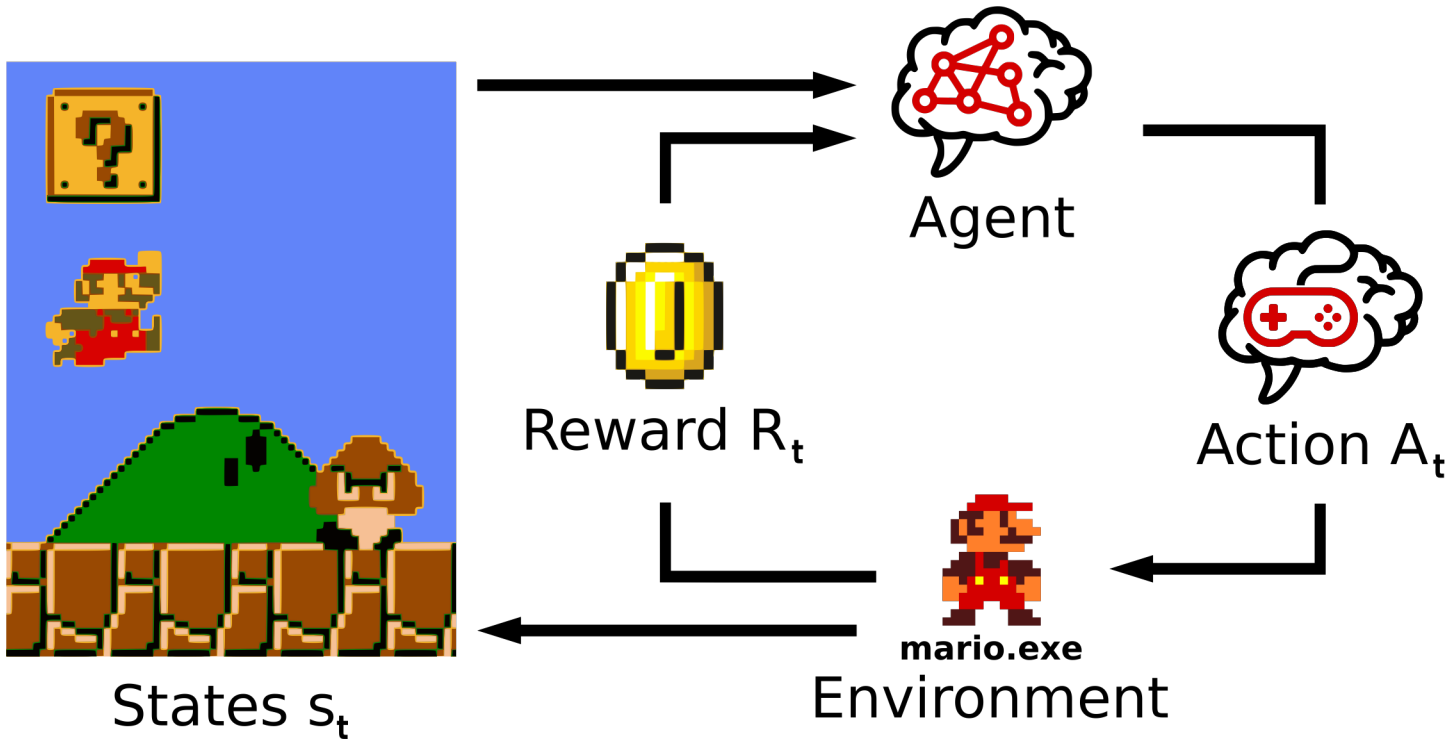

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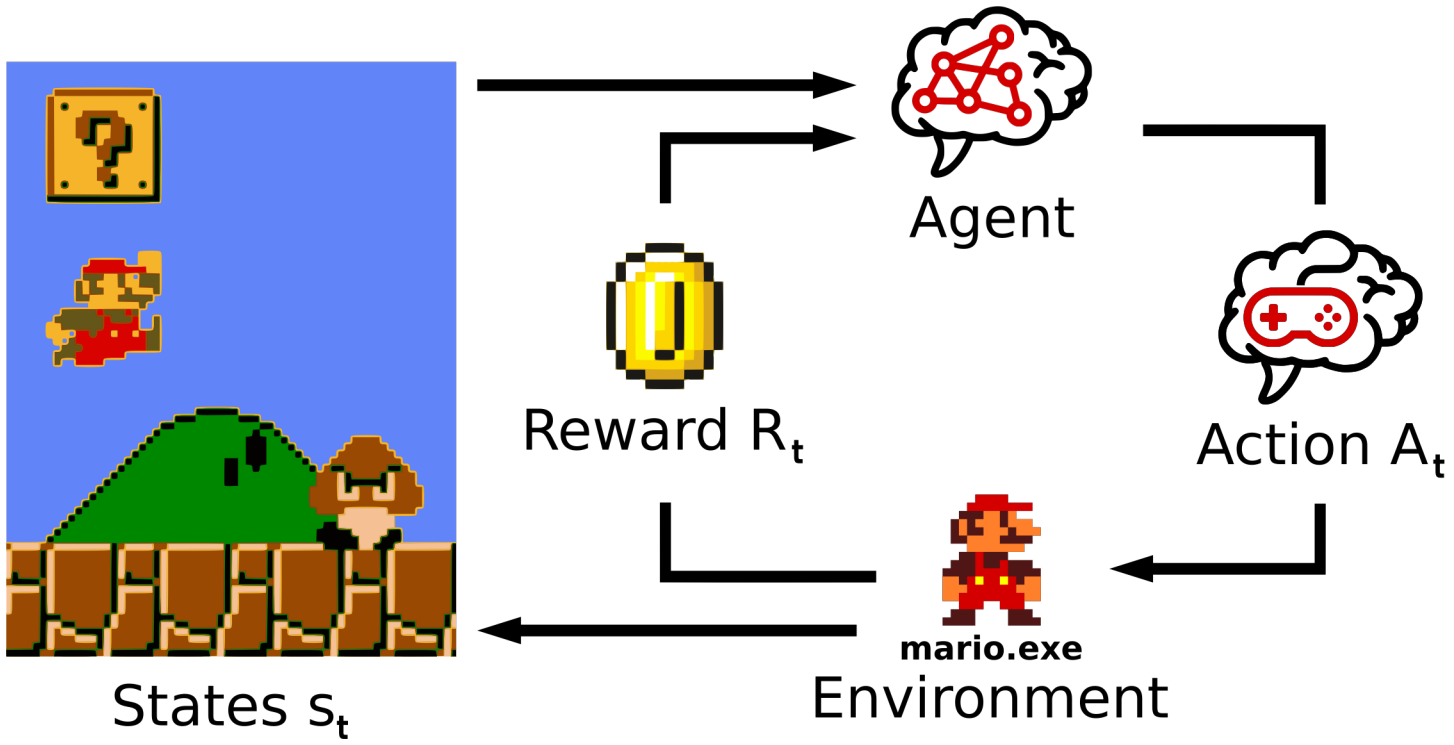

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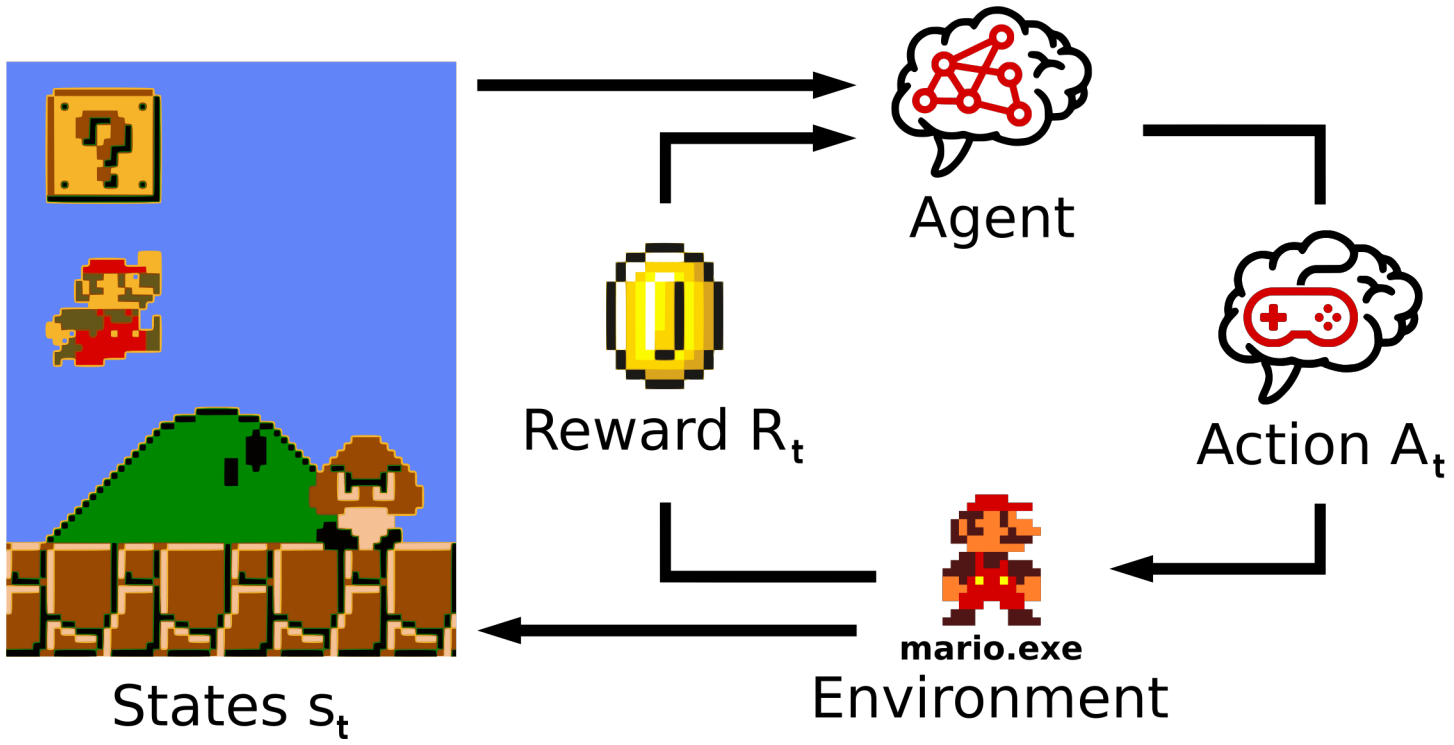

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The interaction within an *episode* leads to a *trajectory* $\boldsymbol{\tau} = (s_0, a_0, r_0, s_1, a_1, r_1, \cdots, s_T)$

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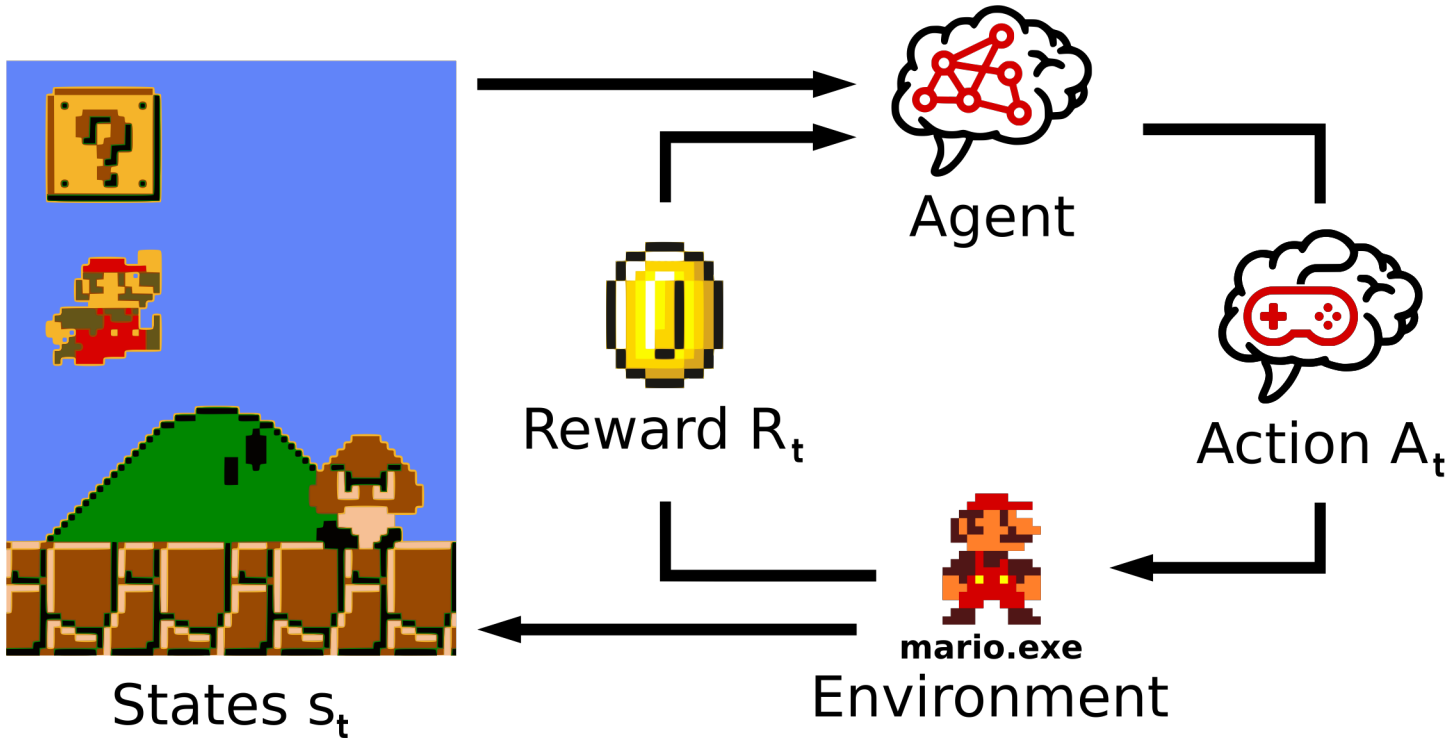
Reinforcement Learning (RL)

As a learning paradigm, RL is different from supervised/unsupervised learning:



- Supervision is scarce, e.g., *reward* is often a scalar
- Supervision is often delayed, e.g., an agent gets the reward after a sequence of actions
- Sequential data is often non-iid, e.g., an agent's current decision would affect the future data distribution

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Markov Decision Process

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A Markov decision process (MDP) is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

Markov Property: The future is independent of the past given the present!

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix $\mathcal{P}^a_{ss'} = \mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a)$
- $\mathcal{R}$ is a reward function $\mathcal{R}^a_s = \mathbb{E}\left[R_{t+1} | S_t = s, A_t = a\right]$
- $\gamma \in [0, 1]$ is a discount factor

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$$\mathcal{P}^a_{ss'} = \mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a)$$

$$\mathcal{R}^a_s = \mathbb{E}\left[R_{t+1} | S_t = s, A_t = a\right]$$

MDP describes an environment where all states are Markov and can be extended to:

- countably infinite states and or action spaces
- continuous state and or action spaces
- continuous time (requires partial differentiable equations)
- partially observable (POMDPs)

Markov Decision Process

Return: the total discounted reward from time t

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

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Policy: the distribution over actions given states

$$\pi(a|s) = \mathbb{P}(A_t = a|S_t = s)$$

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Value (a.k.a., State-Value) function: the expected return starting from state s and then following policy π

$$V_\pi(s) = \mathbb{E}_\pi \left[G_t|S_t = s\right]$$

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Optimal value function

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$$Q_\pi(s,a) = \mathbb{E}_\pi \left[G_t|S_t = s, A_t = a\right]$$

Optimal Q function

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Bellman Equation

Most RL algorithms are based on *Bellman Equation*, which is a recursive formula and has many variations. In particular, for Q-function, we have:

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Most RL algorithms are based on *Bellman Equation*, which is a recursive formula and has many variations.
In particular, for Q-function, we have:

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Proof by induction using *stationary* policies and *homogeneous* Markov chains!

Optimal Bellman Equation

Recall the *optimal Q function is* $Q_*(s,a) = \max_\pi \mathbb{E}_\pi \left[G_t | S_t = s, A_t = a \right]$

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The optimal Bellman equation gives a recursive formula for the optimal Q function:

$$\begin{aligned} Q_*(s,a) &= \max_\pi \mathbb{E}_\pi \left[G_t | S_t = s, A_t = a\right] \\ &= \max_\pi \mathcal{R}_s^a + \gamma \sum_{s',a'} \pi(a'|s') \mathcal{P}_{ss'}^a Q_\pi(s',a') \\ &= \mathcal{R}_s^a + \gamma \max_\pi \sum_{s',a'} \pi(a'|s') \mathcal{P}_{ss'}^a Q_\pi(s',a') \\ &= \mathcal{R}_s^a + \gamma \sum_{s'} \mathcal{P}_{ss'}^a Q_*(s', a_*(s')) \\ &= \mathcal{R}_s^a + \gamma \sum_{s'} \mathcal{P}_{ss'}^a \max_{a'} Q_*(s',a') \end{aligned}$$

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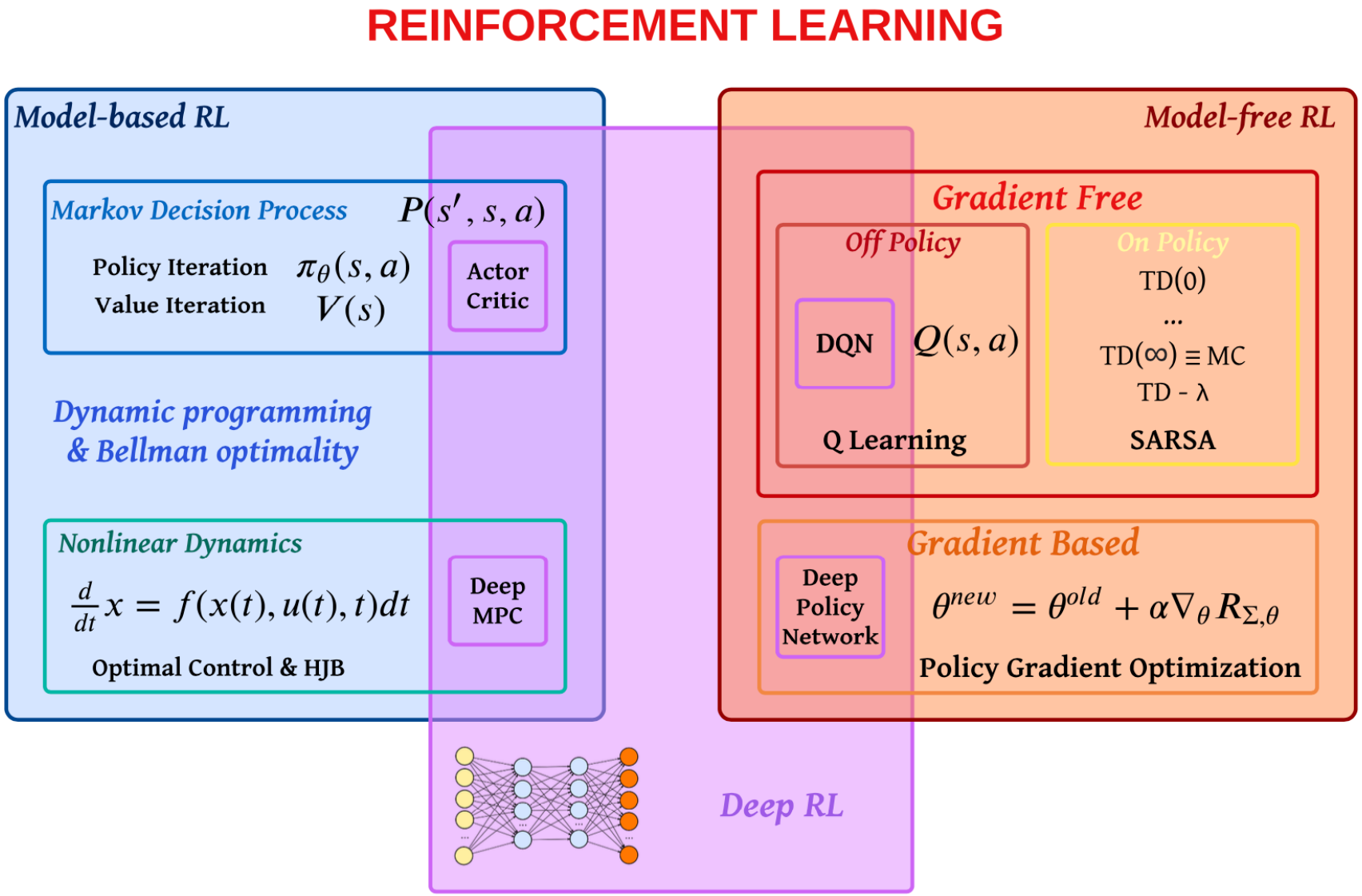
- Reinforcement Learning
  - Overview & Applications
  - Key Concepts
  - Markov Decision Process (MDP) and its Extensions
  - Bellman Equation and its Optimality
  - **RL Taxonomy**
- Deep Reinforcement Learning
  - Q-learning & Deep Q-learning
  - Policy Gradient Methods

RL Taxonomy



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$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta \left[\mathcal{R}_{s_t}^{a_t} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

If this update converges, the Bellman error should reach 0!

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Exploration-exploitation tradeoff: Q learning only learns from the state-action pairs it visits. One often needs some strategy to improve the exploration, e.g., ϵ-greedy policy [11] (choose optimal action based on Q with probability ϵ and choose a random action with probability 1- ϵ).

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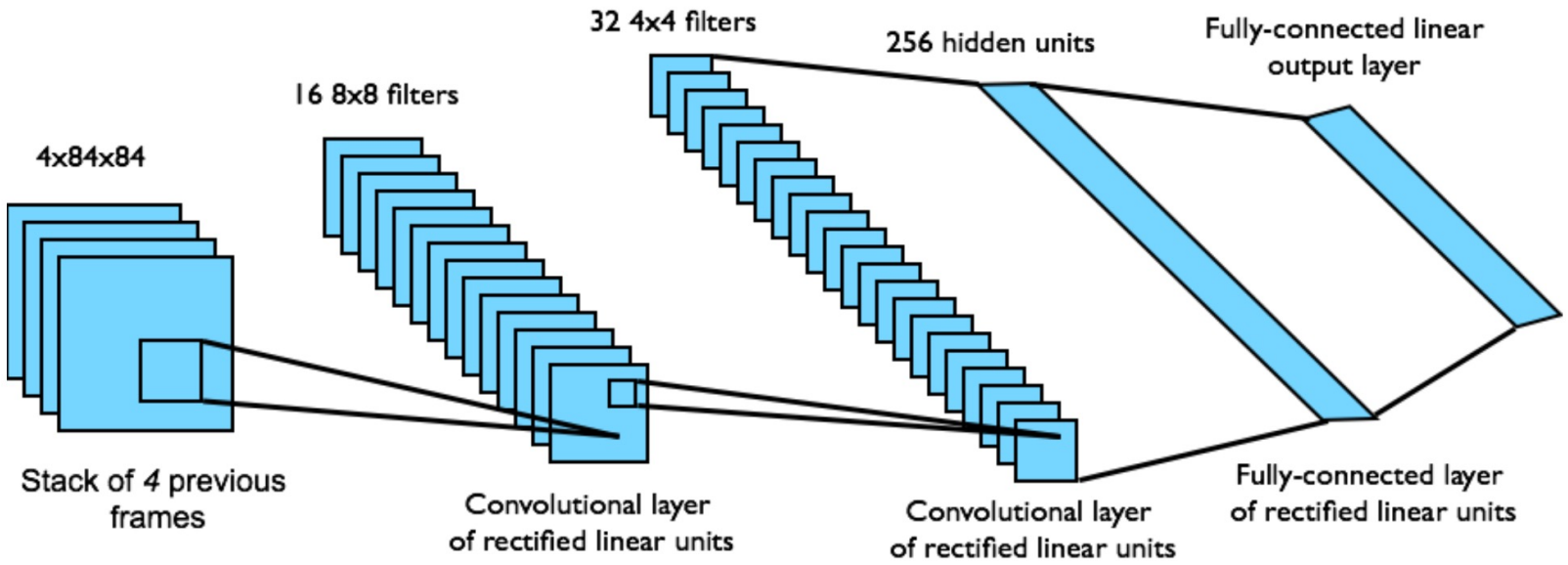
For small state and action spaces, we can represent Q function as a table and learn it. However, for large or infinite spaces, we need to represent it as a parametric function, e.g., a deep neural network!

Deep Q Learning

Approximating Q function with a neural net is a decades-old idea, but DeepMind got it to work really well on Atari games in 2013 (“deep Q-learning”) [12].



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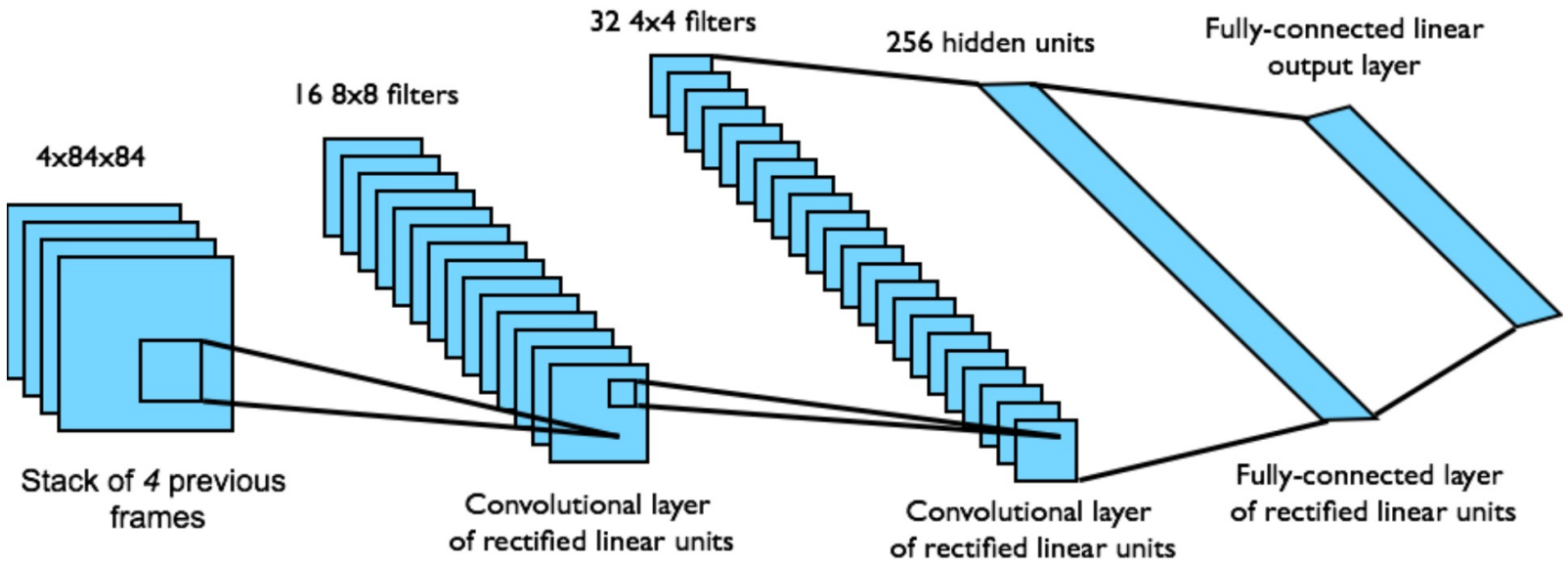

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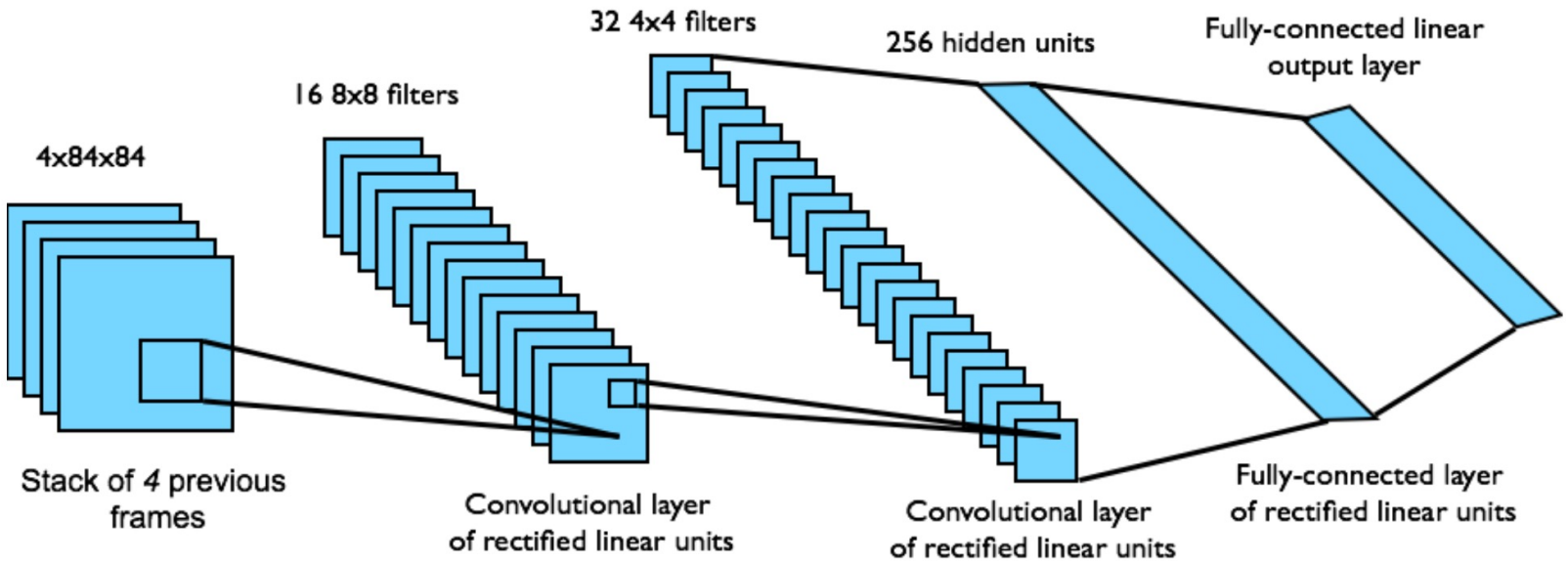

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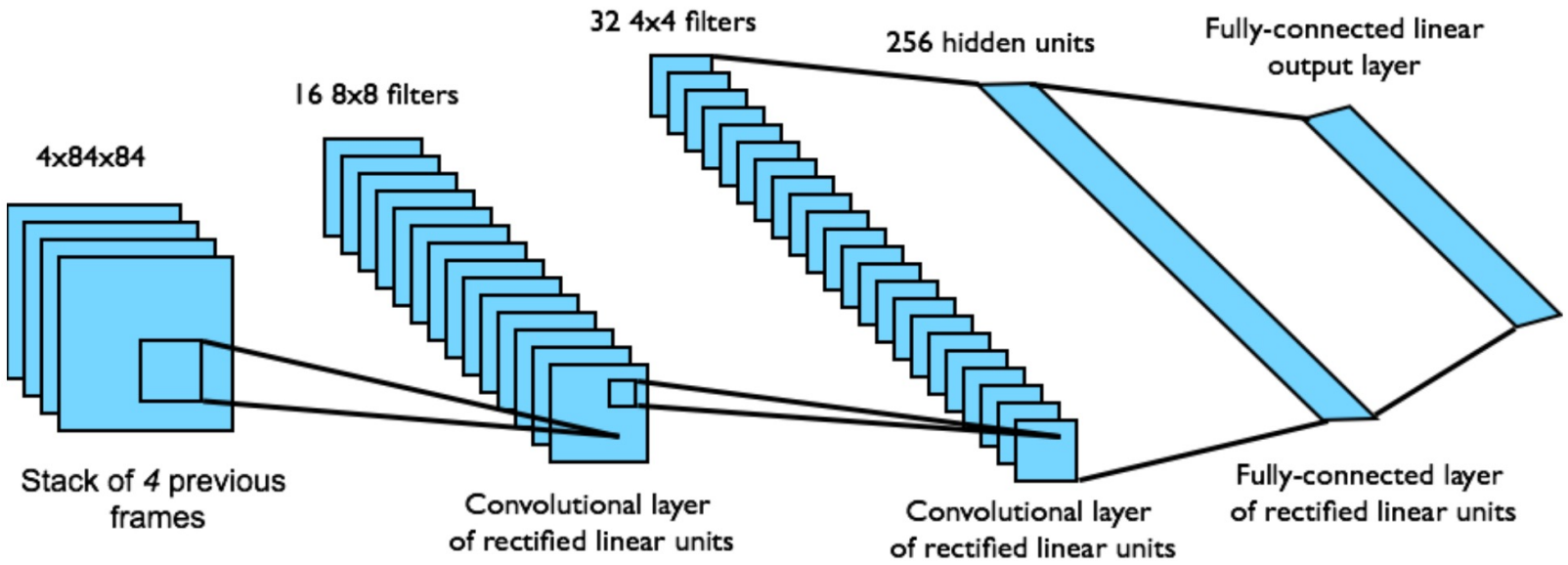
- Compute Q-targets w.r.t. old and fixed parameters $\bar{\theta}$

$$\theta_{t+1} \leftarrow \theta_t + \eta \mathbb{E}_{s_t, a_t, s_{t+1}} \left[\left(\mathcal{R}_{s_t}^{a_t} + \gamma \max_a Q_{\bar{\theta}}(s_{t+1}, a) - Q_{\theta_t}(s_t, a_t) \right) \frac{\partial Q_{\theta_t}(s_t, a_t)}{\partial \theta_t} \right]$$



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In deep Q learning, we parameterize the Q function as a neural network and learn it to minimize Bellman error. A policy is then obtained from Q function, e.g., via ϵ-greedy strategy.

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- Value Based
  - Learnt Value Function
  - Implicit policy (e.g. ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
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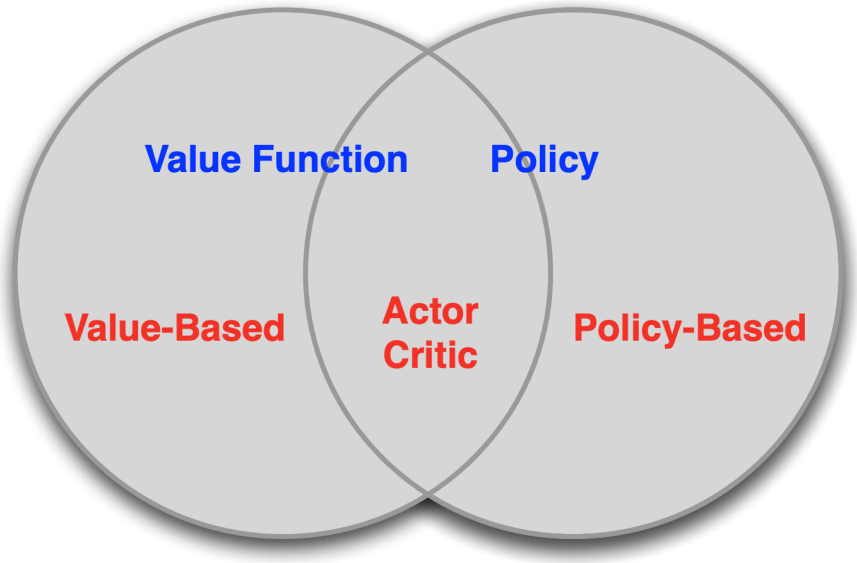

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Advantages:

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Disadvantages:

- Often converge to local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

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In policy based methods, we directly parameterize the policy and learn it to maximize some reward.

Given a trajectory $\boldsymbol{\tau}$, let us consider the simple expected reward:

$$J(\theta) = \mathbb{E}_{\boldsymbol{\tau}}\left[R(\boldsymbol{\tau})\right] = \int \mathbb{P}_{\theta}(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

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Log derivative trick:

$$\nabla_\theta J(\theta) = \nabla_\theta \int \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

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Monte Carlo Approximation!
REINFORCE algorithm [15]

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&= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log\left(\mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta\left(\log \mathbb{P}_0(S) + \sum_{t=1}^{T}\log\pi_\theta(A_t|S_t) + \sum_{t=1}^{T}\log\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(A_t|S_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=1}^{T} r_t^{(i)}\right)
\end{aligned}
$$

No dependence on starting and transition probability of the environment, thus being model-free!

Monte Carlo Approximation!
REINFORCE algorithm [15]

Policy Gradient Methods

Let us substitute $\mathbb{P}_\theta(\boldsymbol{\tau}) = \mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)$ $\quad R(\boldsymbol{\tau}) = \sum_{t=1}^{T} R_t(A_t,S_t)$

We have

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau})\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log\left(\mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta\left(\log \mathbb{P}_0(S) + \sum_{t=1}^{T}\log\pi_\theta(A_t|S_t) + \sum_{t=1}^{T}\log\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(A_t|S_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=1}^{T} r_t^{(i)}\right)
\end{aligned}$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\left(\sum_{t'=t+1}^{T} r_{t'}^{(i)}\right)\right)$$

No dependence on starting and transition probability of the environment, thus being model-free!

Monte Carlo Approximation! REINFORCE algorithm [15]

Reward-to-go version: *my action today can not change rewards in the past*!

Policy Gradient Methods

Let us substitute $\mathbb{P}_\theta(\boldsymbol{\tau}) = \mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)$ $\quad R(\boldsymbol{\tau}) = \sum_{t=1}^{T} R_t(A_t,S_t)$

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&= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta\left(\log \mathbb{P}_0(S) + \sum_{t=1}^{T}\log\pi_\theta(A_t|S_t) + \sum_{t=1}^{T}\log\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&= \mathbb{E}_{\boldsymbol{\tau}}\left[\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(A_t|S_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right] \\
&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=1}^{T} r_t^{(i)}\right)
\end{aligned}$$

No dependence on starting and transition probability of the environment, thus being model-free!

Monte Carlo Approximation! REINFORCE algorithm [15]

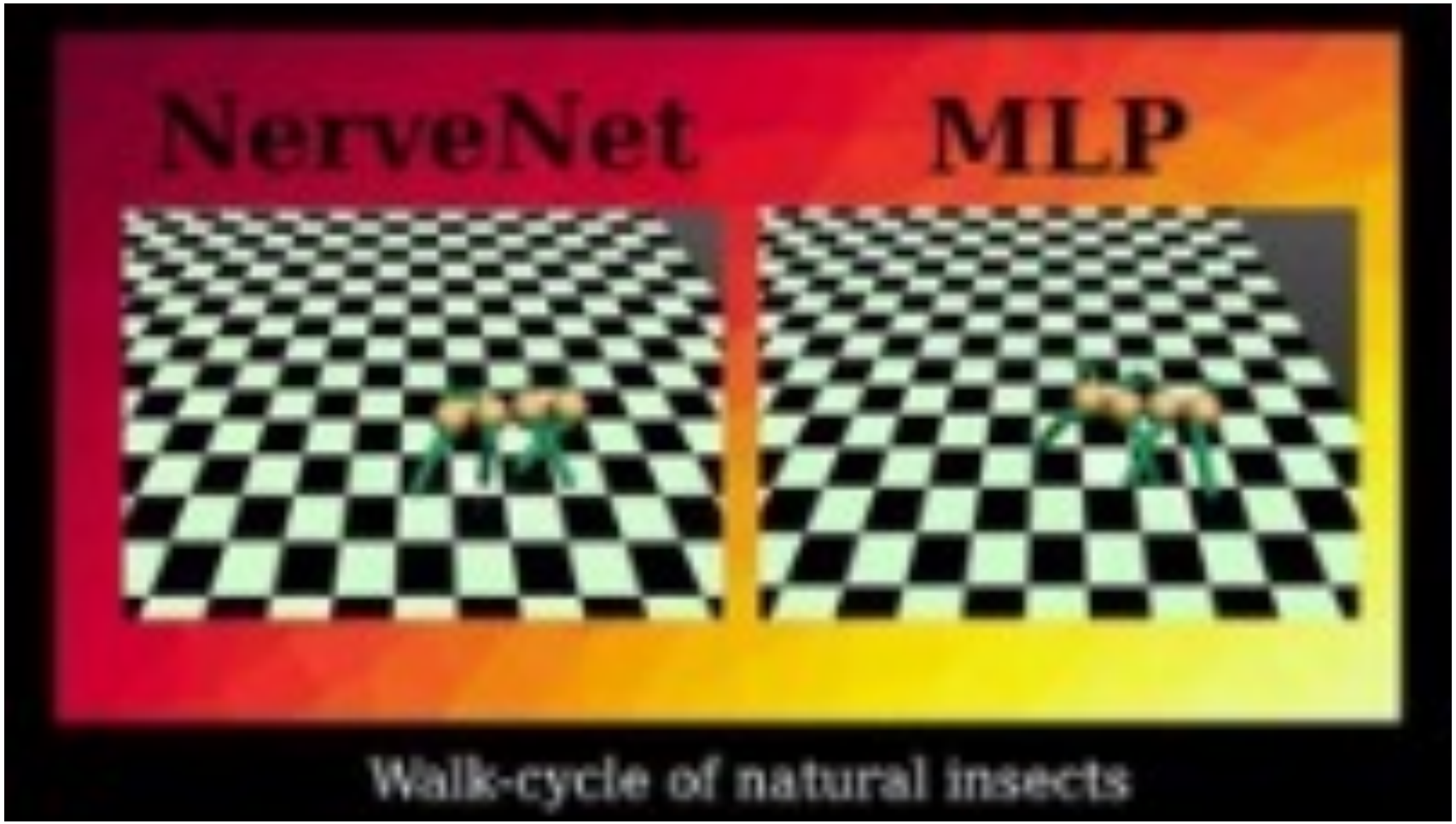
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\left(\sum_{t'=t+1}^{T} r_{t'}^{(i)}\right)\right)$$

Reward-to-go version: *my action today can not change rewards in the past*!

One often uses control variate method to reduce the high variance of policy gradients.

Demo

Simulated Continuous Control in Mujoco using PPO [16] (an advanced policy gradient method) and graph neural networks [17]:



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Questions?