CPEN 455: Deep Learning

Lecture 5: Convolutional Neural Networks II

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University of British Columbia Winter, Term 2, 2024

Outline

- Invariance & Equivariance
- Convolution
 - 1D Convolution
 - Matrix Multiplication Views
 - Translation Equivariance
 - 2D Convolution
- Convolution Variants
 - Transposed Convolution
 - Dilated Convolution
 - Grouped Convolution
 - Separable Convolution
- Pooling
- Example Architectures

We know convolution can reduce the input size, e.g., with stride > 1. Can any convolution operator enlarge the input size?

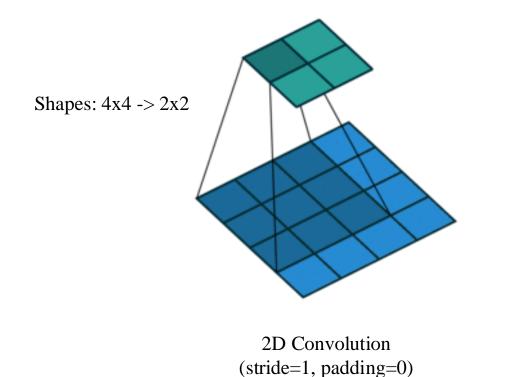
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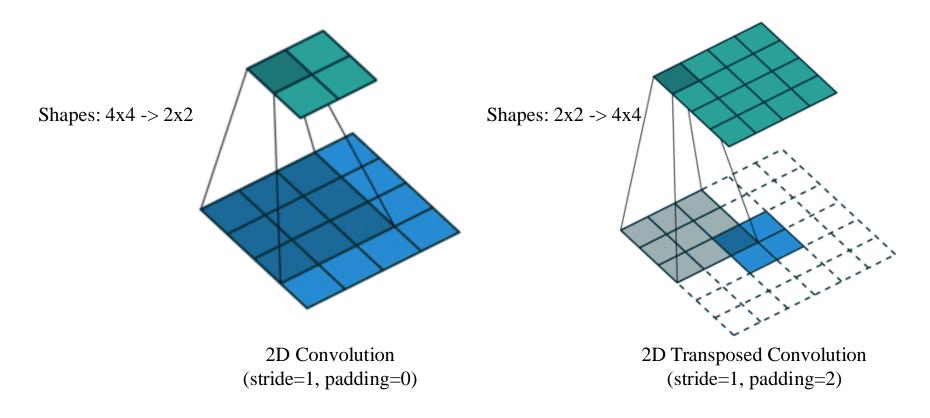
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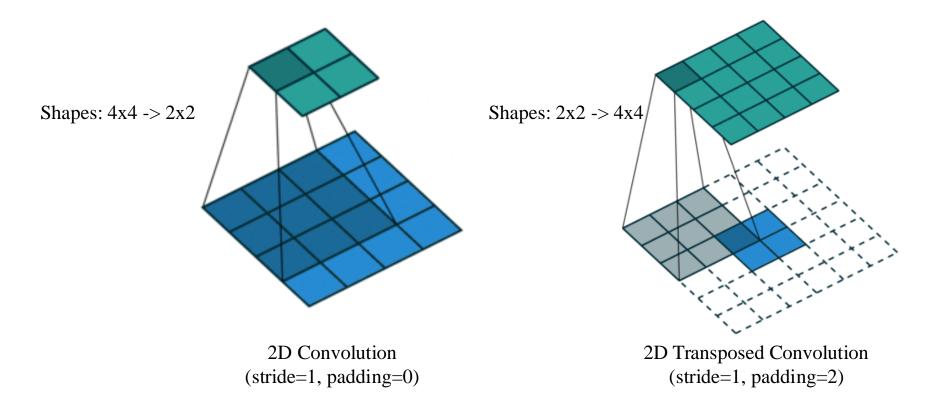


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Suppose we have a 2D convolution (3x3 kernel):

• Convolution and its transposed version are mutually inverse only w.r.t. shapes of input and output, but not w.r.t. values of input and output!

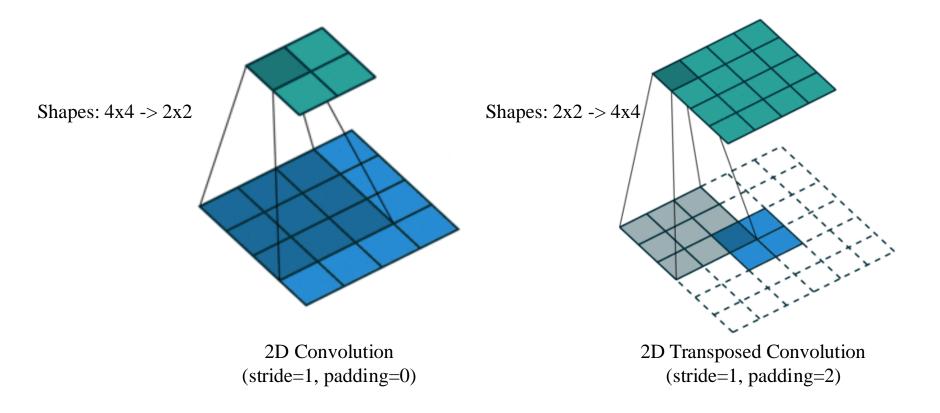


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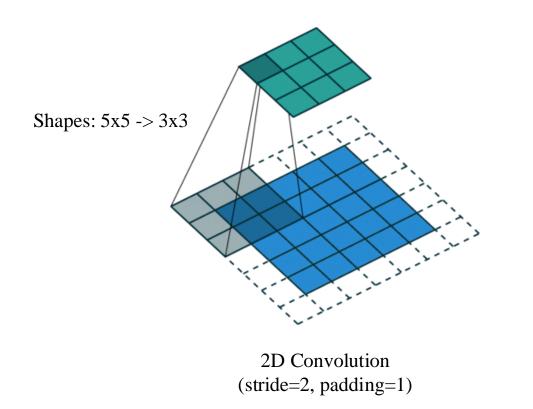
- Convolution and its transposed version are mutually inverse only w.r.t. shapes of input and output, but not w.r.t. values of input and output!
- Convolution and deconvolution are mutually inverse w.r.t. values of input and output!



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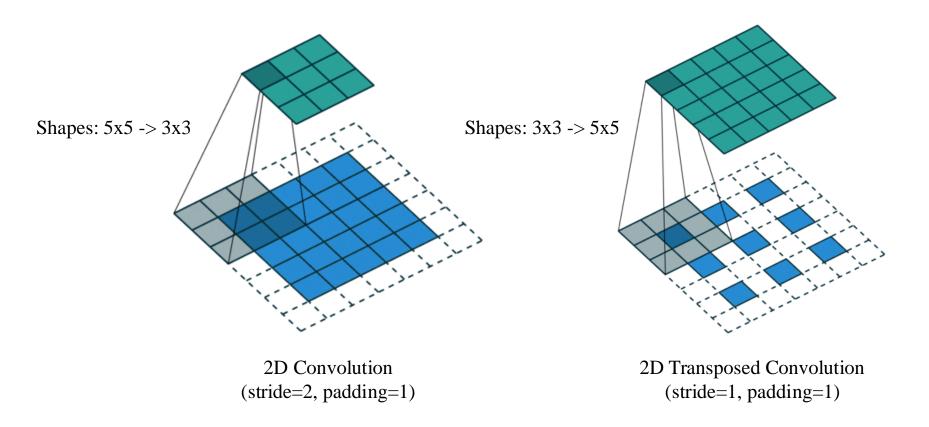
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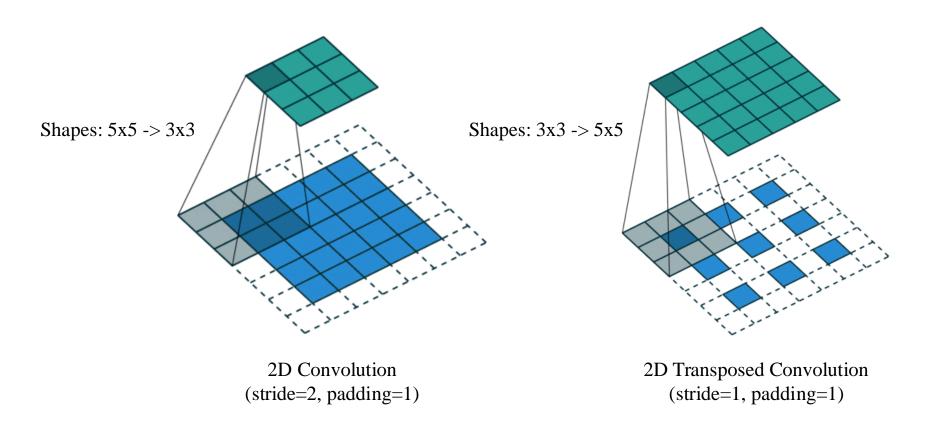


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Try another example of 2D convolution (3x3 kernel):

Transposed convolution is also known as fractionally strided convolution!

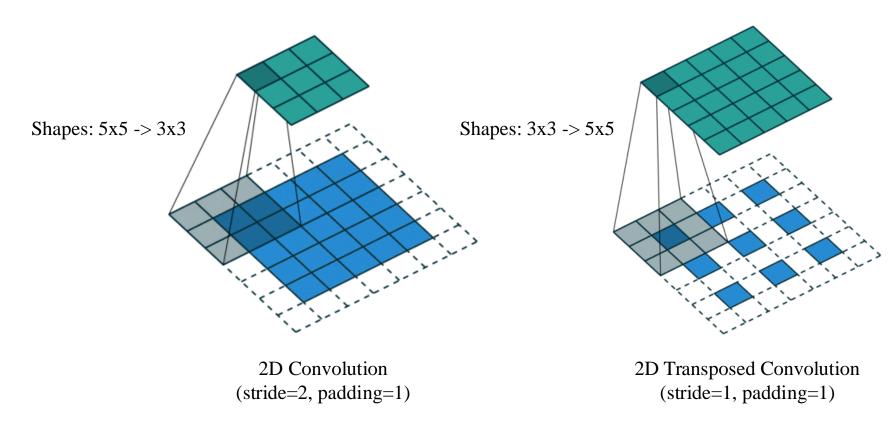


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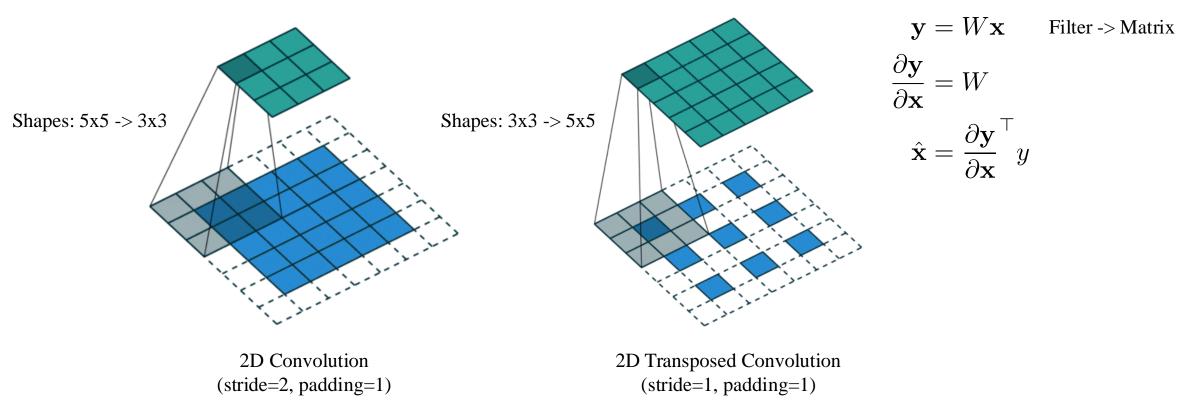


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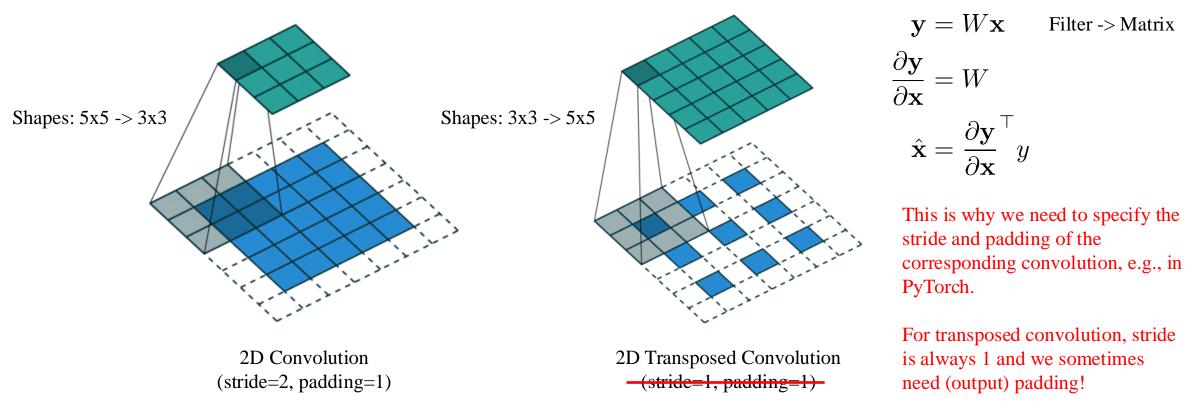


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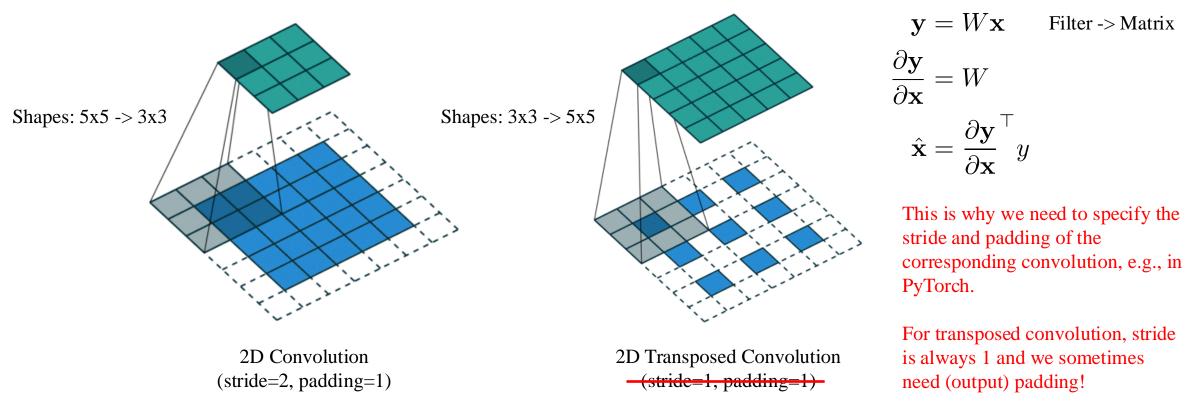


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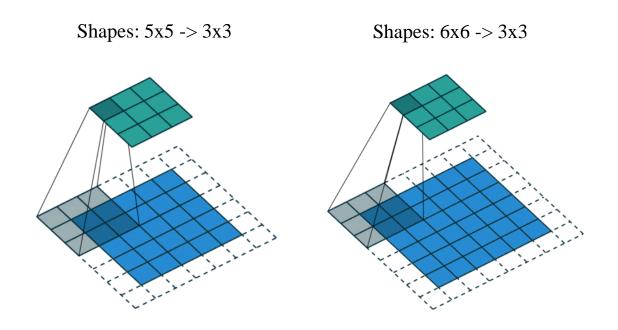
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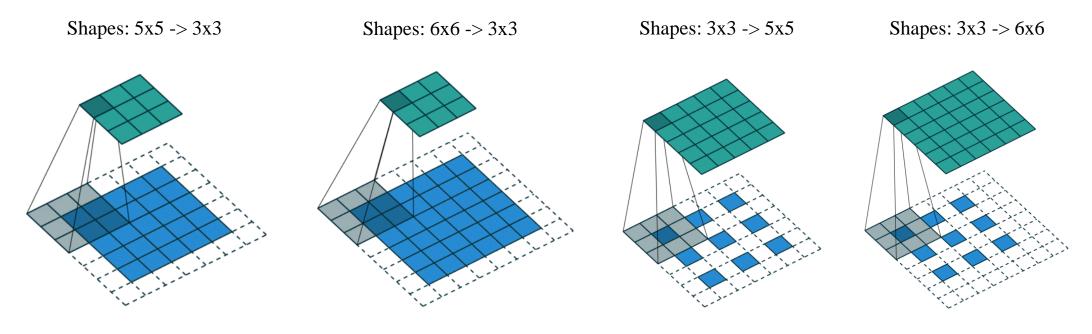


The gradients of the following two convolutions have the same shape in im2patch (data-> toeplitz matrix) implementation.



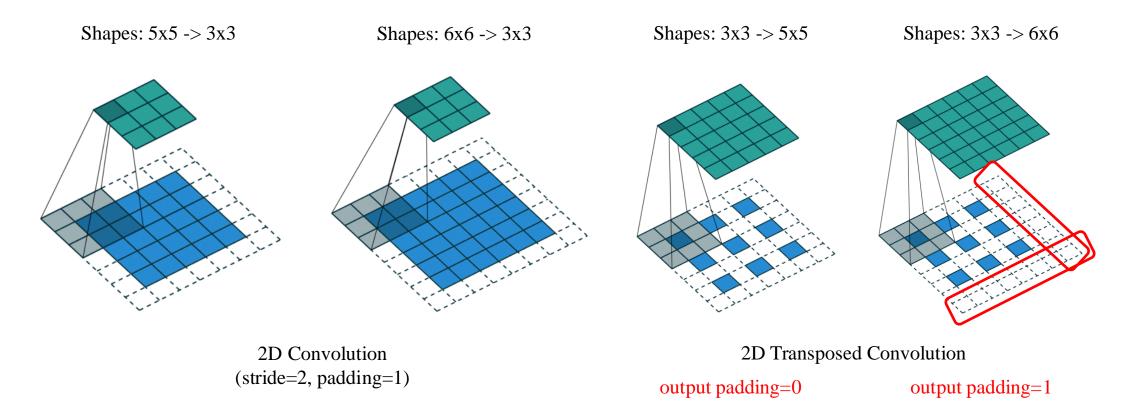
2D Convolution (stride=2, padding=1)

The gradients of the following two convolutions have the same shape in im2patch (data-> toeplitz matrix) implementation. To distinguish them and output correct shapes in their transposed convolutions, we add output padding on one side in the 2nd case.



2D Convolution (stride=2, padding=1) 2D Transposed Convolution

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Take the API in PyTorch as an example

CLASS torch.nn.ConvTranspose2d(in_channels, out_channels, kernel_size, stride=1, padding=0, output_padding=0, groups=1, bias=True, dilation=1, padding_mode='zeros', device=None, dtype=None) [SOURCE]

Applies a 2D transposed convolution operator over an input image composed of several input planes.

This module can be seen as the gradient of Conv2d with respect to its input. It is also known as a fractionally-strided convolution or a deconvolution (although it is not an actual deconvolution operation as it does not compute a true inverse of convolution). For more information, see the visualizations here and the Deconvolutional Networks paper.

This module supports TensorFloat32.

On certain ROCm devices, when using float16 inputs this module will use different precision for backward.

- stride controls the stride for the cross-correlation.
- padding controls the amount of implicit zero padding on both sides for dilation * (kernel_size 1) padding number of points. See note below for details.
- output_padding controls the additional size added to one side of the output shape. See note below for details.

stride of convolution, not the stride of transposed convolution!

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We know the kernel size decides what elements are used in convolution at one location. Can we enlarge the kernel size without increasing the number of parameters?

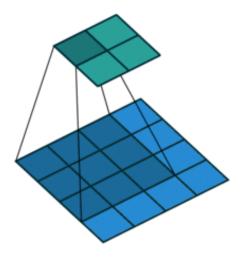
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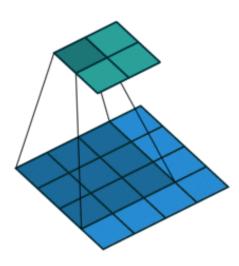


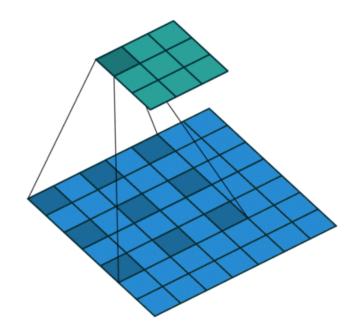
2D Convolution (stride=1, padding=0)

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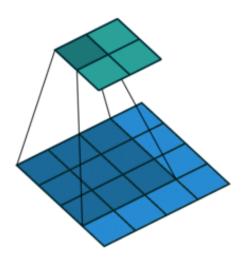


2D Convolution (stride=1, padding=0) 2D Dilated Convolution (stride=1, padding=0, dilation=2)

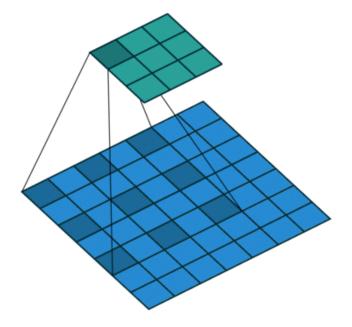
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By using dilated kernels, we effectively increase the receptive field (the region of input that affects the output)!



2D Convolution (stride=1, padding=0) 2D Dilated Convolution (stride=1, padding=0, dilation=2)

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Can we maintain the same shaped input and output in convolution with fewer number of parameters?

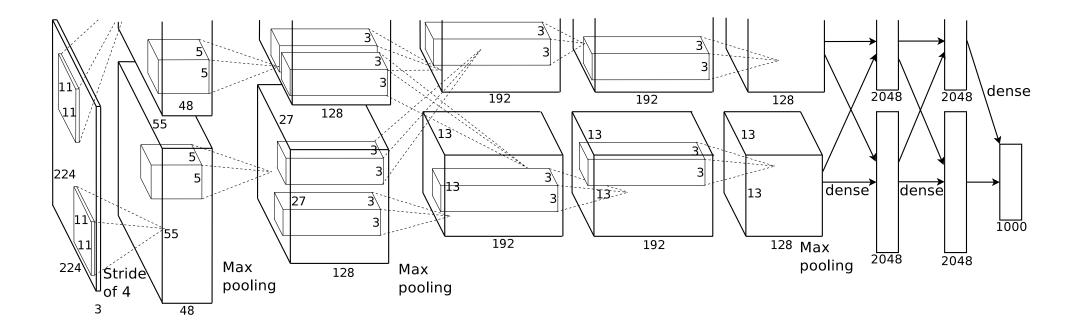
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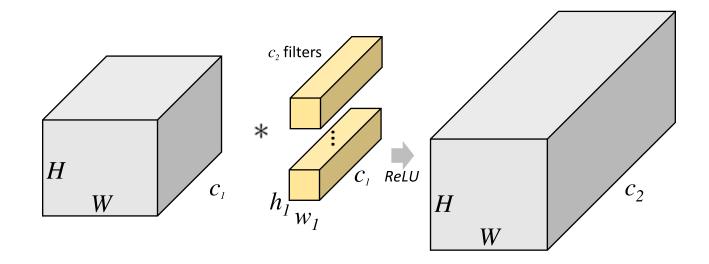
It was first proposed in AlexNet [2]:



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Suppose we have a convolution layer applied to input (shape $H \times W \times c_1$):

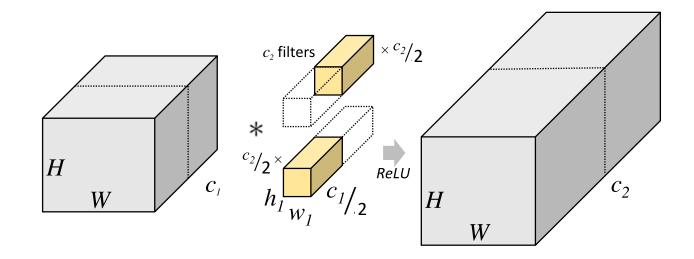


We have c_2 filters with kernel size $h_1 \times w_1 \times c_1$

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Now we switch to a grouped (# groups=2) convolution layer applied to the same input (shape $H \times W \times c_1$):

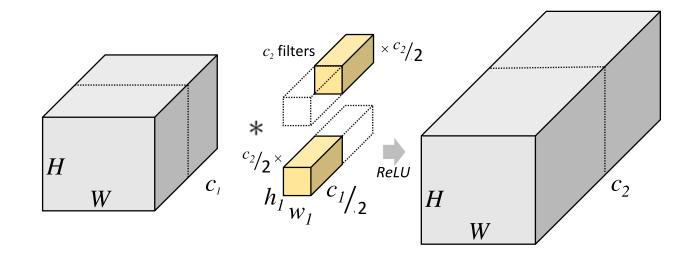


We have 2 groups of filters, and the total number of parameters is the same as a single filter before!

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Generalize it to multi-groups by yourself!

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Separable Convolution

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Let us look at a 3x3 convolutional kernel:

$$\begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

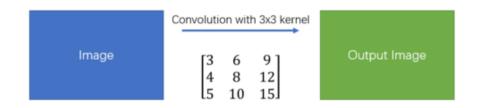
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Simple Convolution



Spatial Separable Convolution



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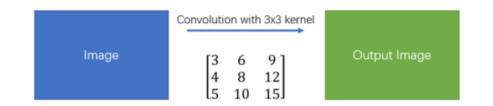
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Simple Convolution

Spatial separable kernels are rank one and can not represent full-rank kernels, thus being limited in terms of expressiveness!



Spatial Separable Convolution



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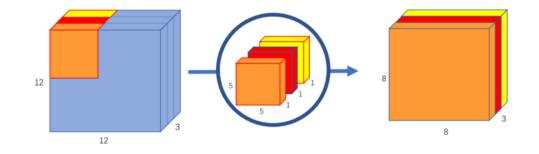
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• Depthwise spatial convolution



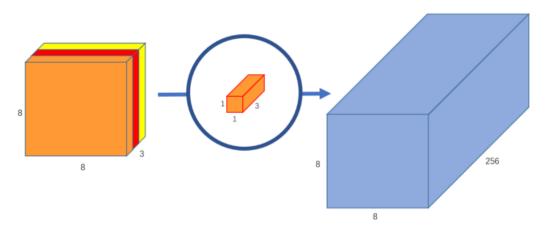
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- Pointwise 1x1 convolution





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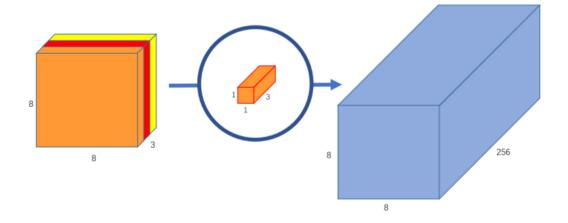
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It is a separable convolution: spatial \times depth (channel)!



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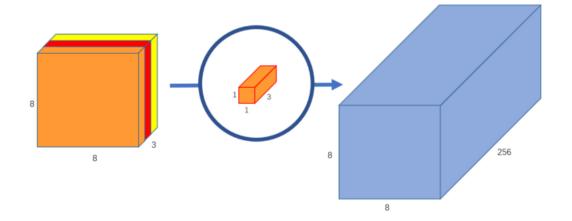
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 $12 \underbrace{12}_{12}$

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Work out the numbers of parameters and operations, you will find it saves both!



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Pooling

A similar idea as convolution except that you replace *weighted sum* operator with some pooling operator (e.g., *max*, mean)

2 X 2 Max Pooling with Stride 2

1	0	3	5	
3	4	2	2	<u>`</u>
1	3	3	9	
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2 X 2 Mean Pooling with Stride 2

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2	3
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L		0	3	5]		
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L		043	3 2 3	5 2 9		2	3

2 X 2 Mean Pooling with Stride 2

Pooling gives you permutation-invariance!

8

4

7

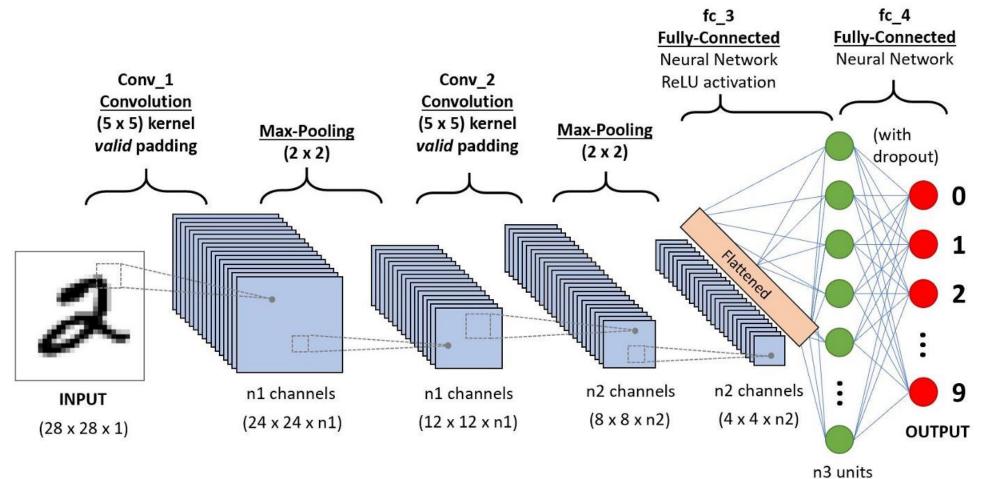
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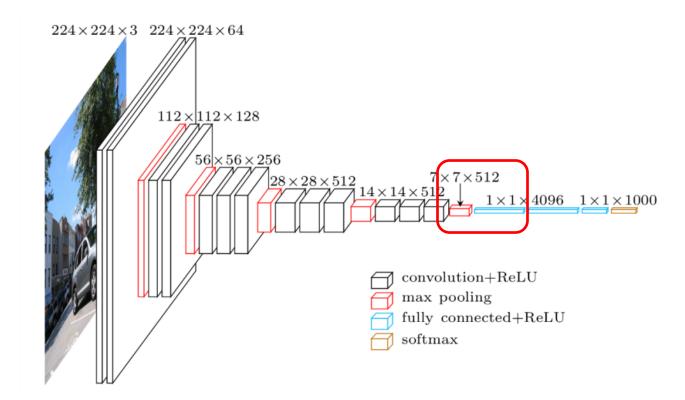
Convolutional Neural Networks (CNNs)

Let us look at an example CNN:



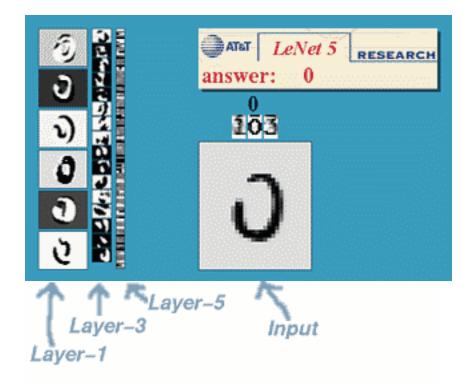
Translation/Shift Invariance

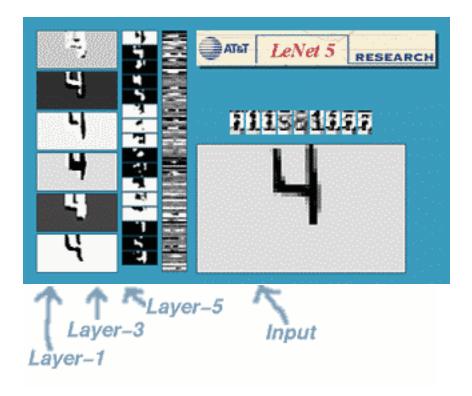
Suppose background does not change and one only shifts the foreground object, pooling gives you shift-invariance!



Translation/Shift Equivariance Invariance

Yann LeCun's LeNet Demo:



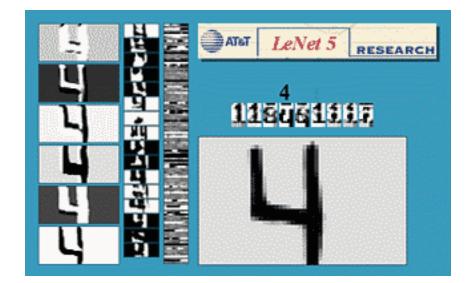


More on Invariance & Equivariance

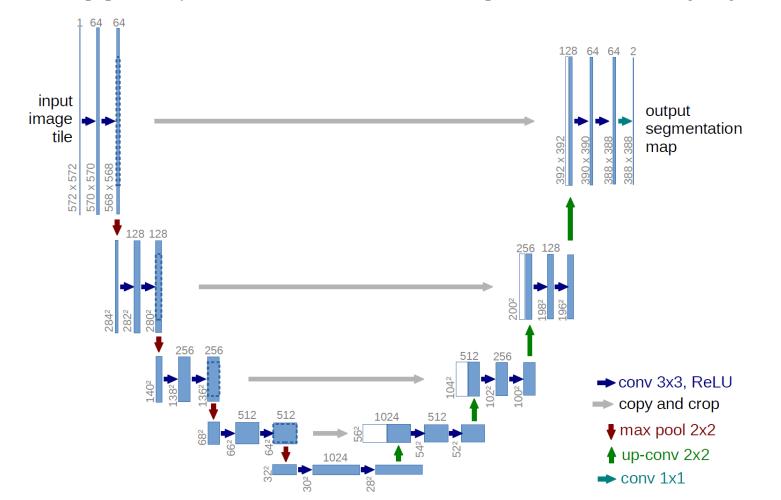
What about other transformations, e.g., scaling, 2D/3D rotations?

Vanilla CNNs do not have such properties. One can add data augmentation to make the model approximately have them.

One can also design CNN architectures, e.g., spherical CNNs (rotation equivariant), that are guaranteed to have such properties [9].

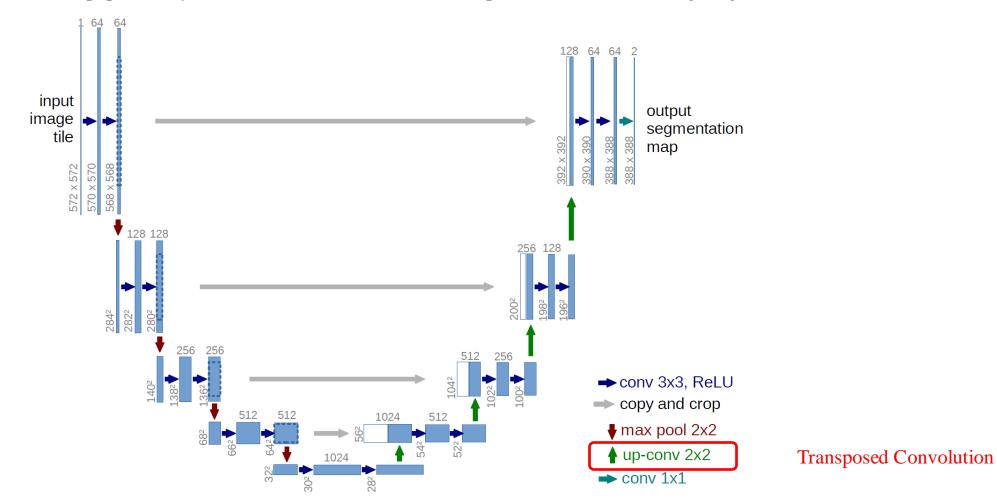


U-Net



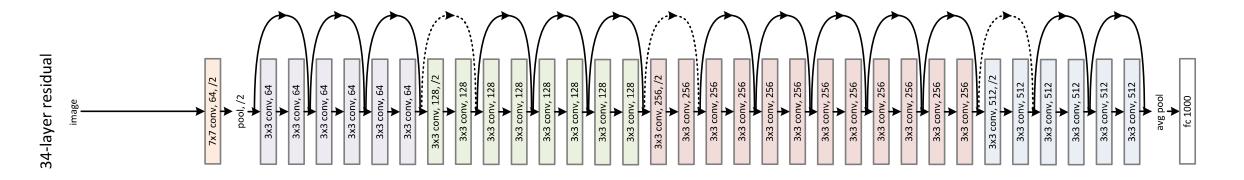
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U-Net

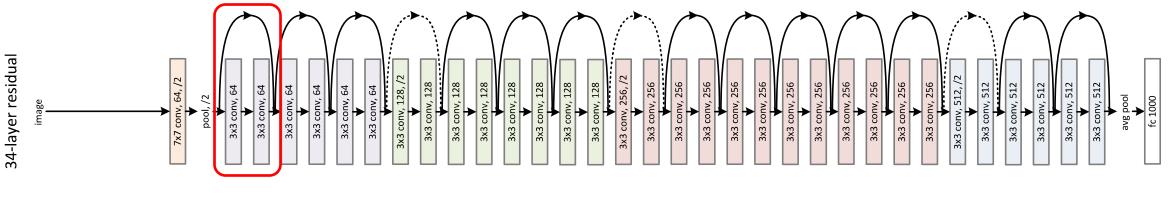


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ResNet [11] is a popular fully-convolutional CNN architecture for pixel-level tasks like image segmentation.

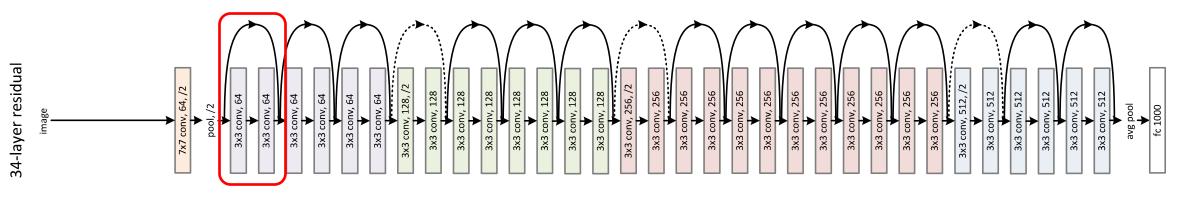


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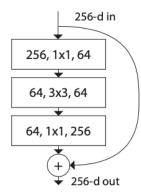
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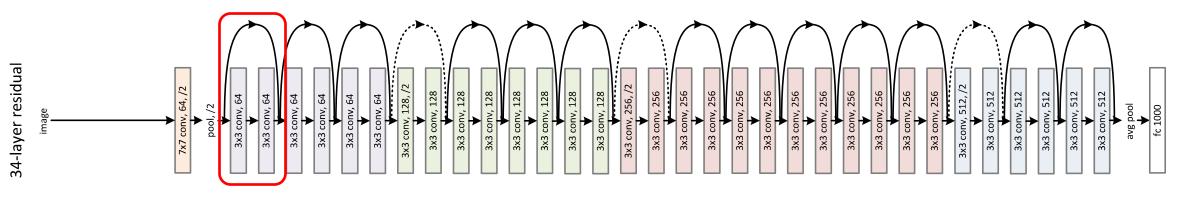


Residual block with skip connection

Build deeper ones (e.g., ResNet-50, ResNet-101) by replacing it with the bottleneck structure!



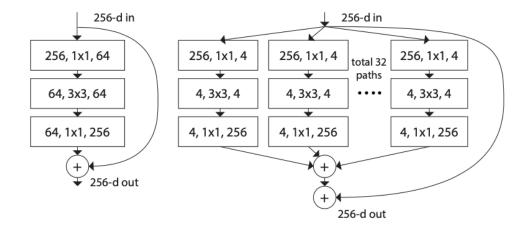
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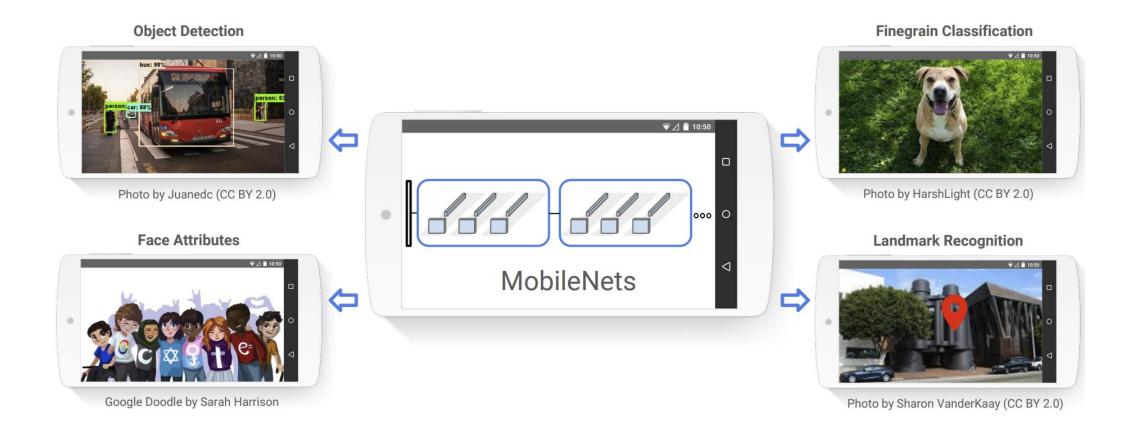
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ResNeXt [12] replaces it with aggregated transformations (similar to grouped convolution but with shared input)!



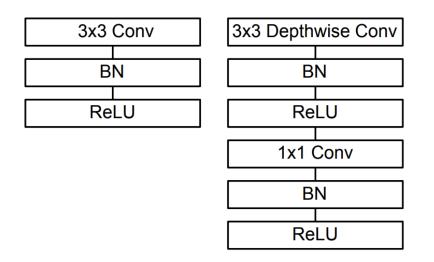
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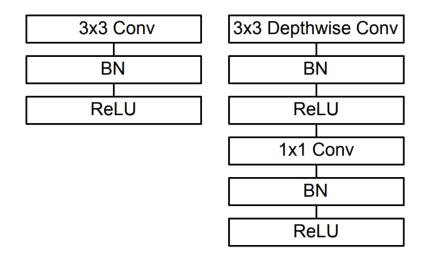
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Replace the vanilla conv layer with depthwise separable convolutional layer

Table 1. MobileNet Body Architecture				
Type / Stride	Filter Shape	Input Size		
Conv / s2	$3 \times 3 \times 3 \times 32$	$224 \times 224 \times 3$		
Conv dw / s1	$3 \times 3 \times 32$ dw	$112 \times 112 \times 32$		
Conv / s1	$1 \times 1 \times 32 \times 64$	$112 \times 112 \times 32$		
Conv dw / s2	$3 \times 3 \times 64$ dw	$112 \times 112 \times 64$		
Conv / s1	$1\times1\times64\times128$	$56 \times 56 \times 64$		
Conv dw / s1	$3 \times 3 \times 128 \text{ dw}$	$56 \times 56 \times 128$		
Conv / s1	$1\times1\times128\times128$	$56 \times 56 \times 128$		
Conv dw / s2	$3 \times 3 \times 128 \text{ dw}$	$56 \times 56 \times 128$		
Conv / s1	$1\times1\times128\times256$	$28 \times 28 \times 128$		
Conv dw / s1	$3 \times 3 \times 256$ dw	$28 \times 28 \times 256$		
Conv / s1	$1\times1\times256\times256$	$28 \times 28 \times 256$		
Conv dw / s2	$3 \times 3 \times 256 \text{ dw}$	$28 \times 28 \times 256$		
Conv / s1	$1\times1\times256\times512$	$14\times14\times256$		
$5 \times Conv dw / s1$	3 imes 3 imes 512 dw	$14 \times 14 \times 512$		
$^{\circ}$ Conv / s1	$1\times1\times512\times512$	$14\times14\times512$		
Conv dw / s2	3 imes 3 imes 512 dw	$14 \times 14 \times 512$		
Conv / s1	$1\times1\times512\times1024$	$7 \times 7 \times 512$		
Conv dw / s2	$3 \times 3 \times 1024 \mathrm{dw}$	$7 \times 7 \times 1024$		
Conv / s1	$1 \times 1 \times 1024 \times 1024$	$7 \times 7 \times 1024$		
Avg Pool / s1	Pool 7×7	$7 \times 7 \times 1024$		
FC / s1	1024×1000	$1 \times 1 \times 1024$		
Softmax / s1	Classifier	$1 \times 1 \times 1000$		

Table 1 Mabile Not Dady Architect

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Questions?