CPEN 455: Deep Learning

Lecture 12: Deep Reinforcement Learning Part II

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Winter, Term 2, 2024

Outline

- Reinforcement Learning (RL) I
  - Overview & Applications
  - RL Formulation & Taxonomy
  - Markov Decision Process (MDP)
  - Bellman Equations
  - Solving RL problems using the Bellman Equations
- Reinforcement Learning (RL) II
  - Model-Free RL: (Deep) Q-learning
  - Model-Free RL: Policy Gradient & Actor-Critic
  - Model-Based RL: Dyna-Q

RL Taxonomy: Recap

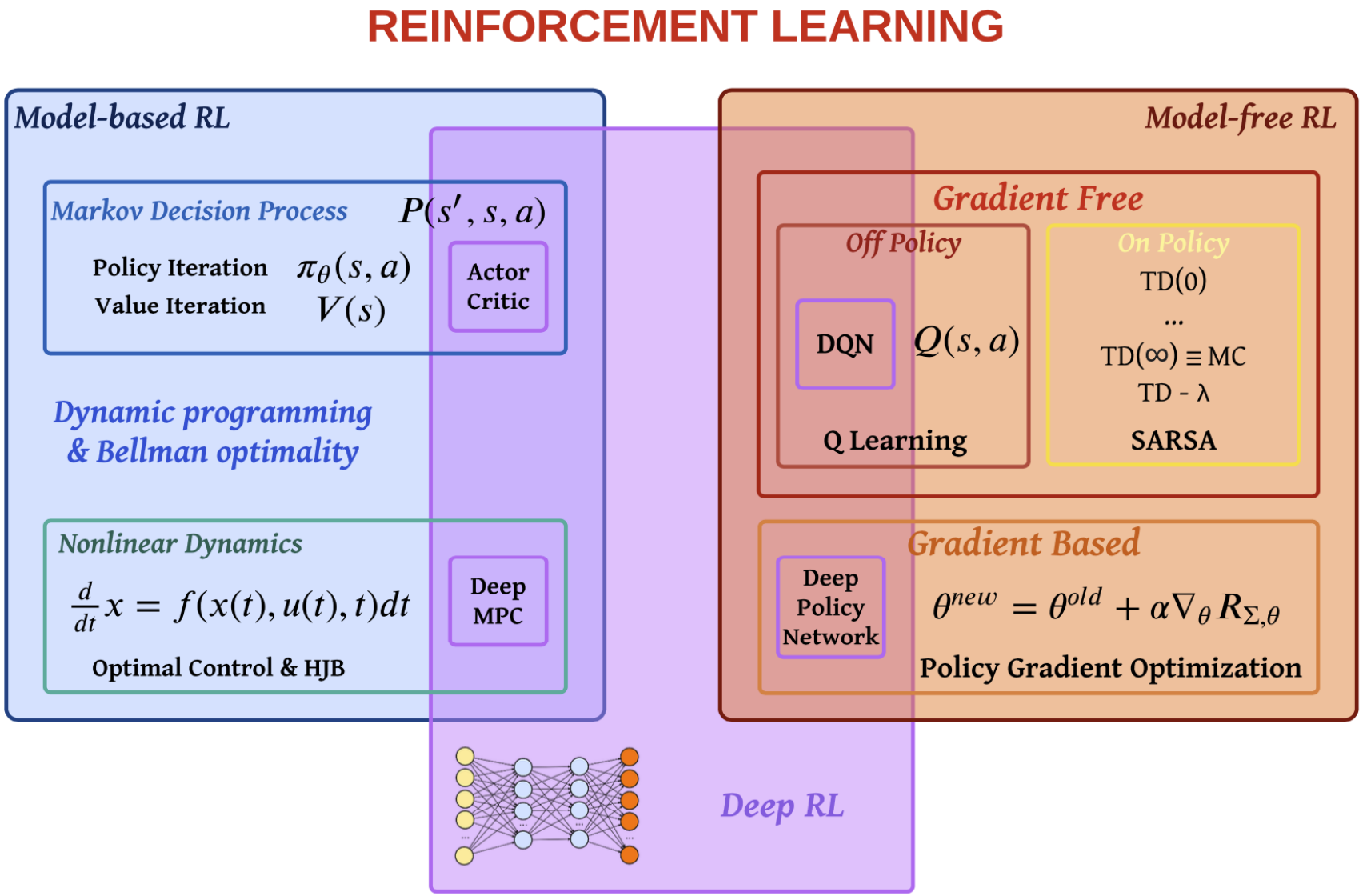

Image Credit: [1]

RL Taxonomy: Recap

Category	Subcategory	Examples	Does It Learn a Model?
Model-Free RL	**Value-Based**	Q-Learning, SARSA, DQN, TD-Learning	❌ No
	Policy-Based	REINFORCE, PPO, TRPO, SAC	❌ No
	Actor-Critic	A2C, A3C, DDPG, TD3	❌ No
Model-Based RL	**Explicit Model Learning**	Dyna-Q, AlphaZero, World Models	✅ Yes
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Model-Free RL

- Value Based
  - Learnt Value Function
  - Implicit policy (e.g. ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
  - Learnt Value Function
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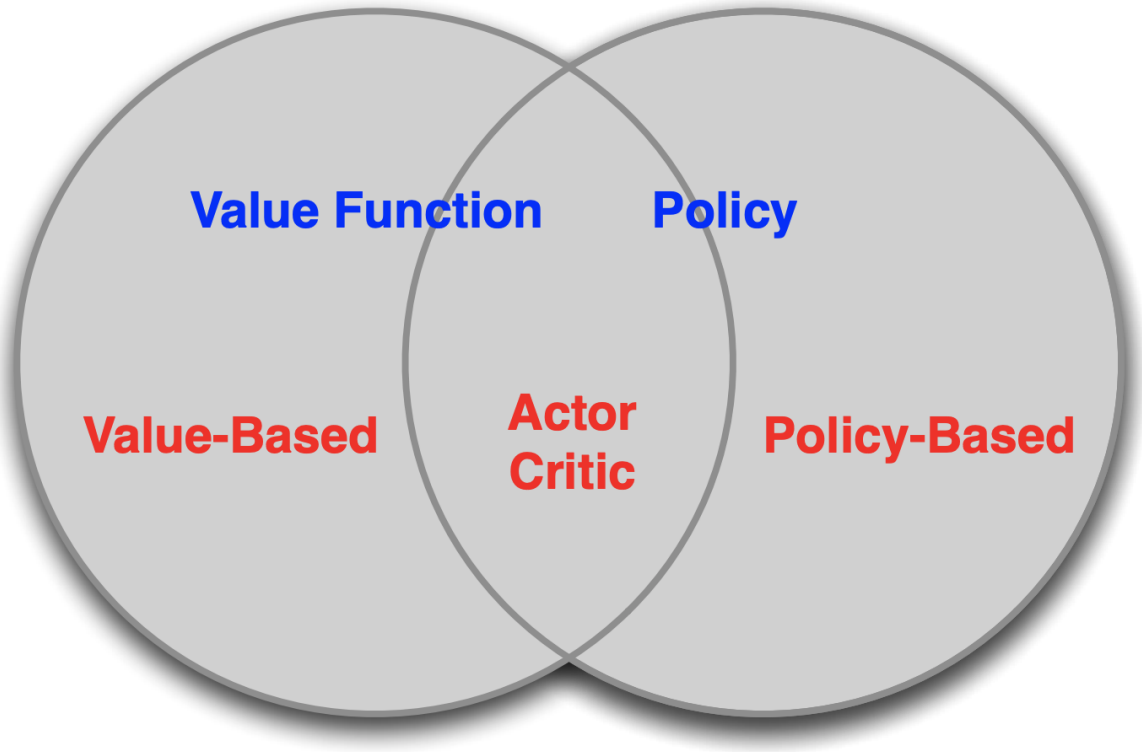
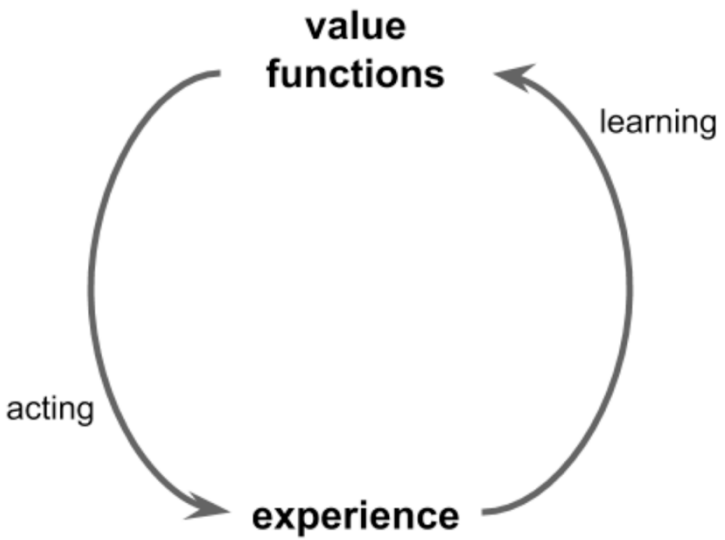


Image Credit: [2]

Model-Based RL



Model-Free RL

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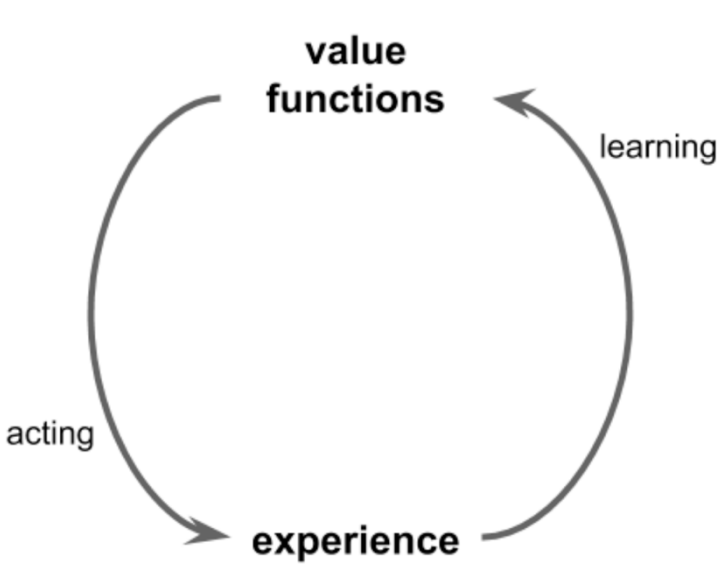

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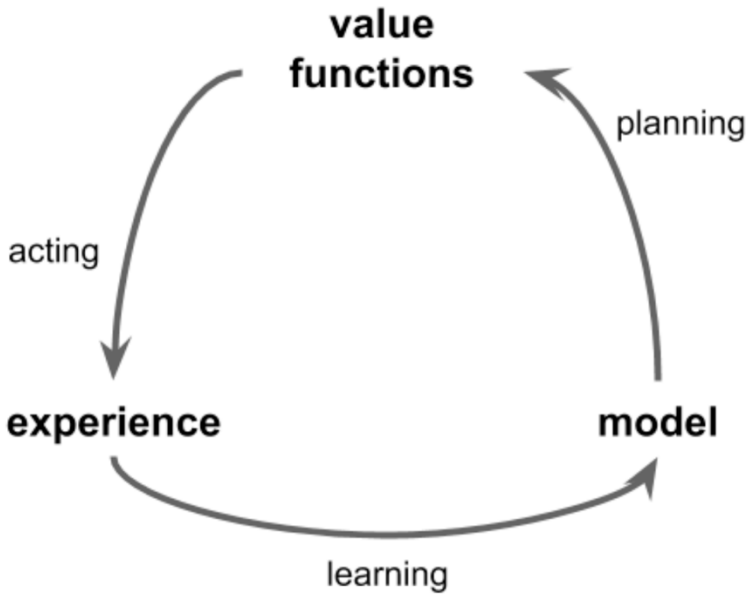

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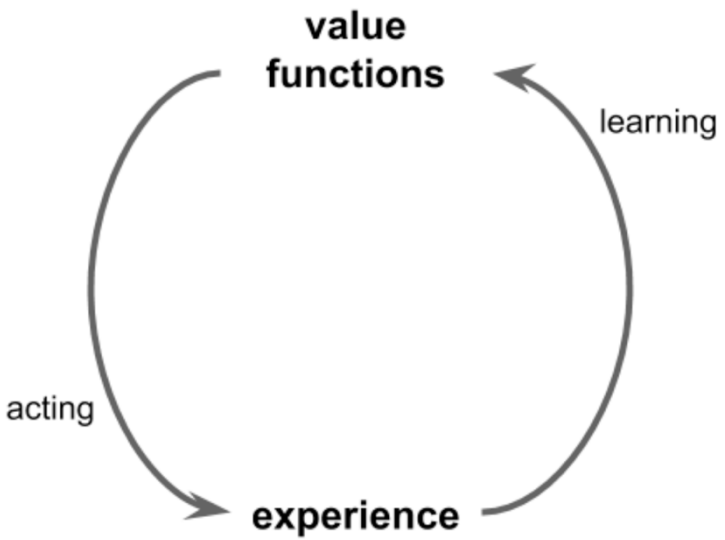

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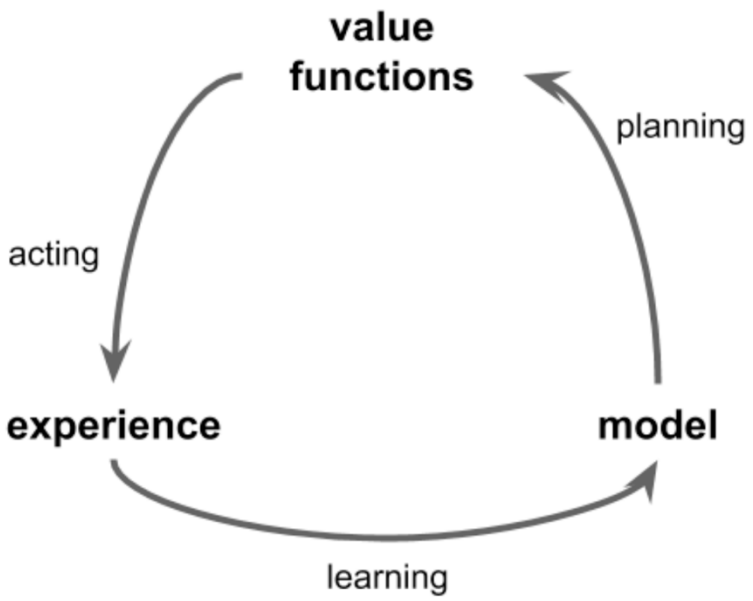

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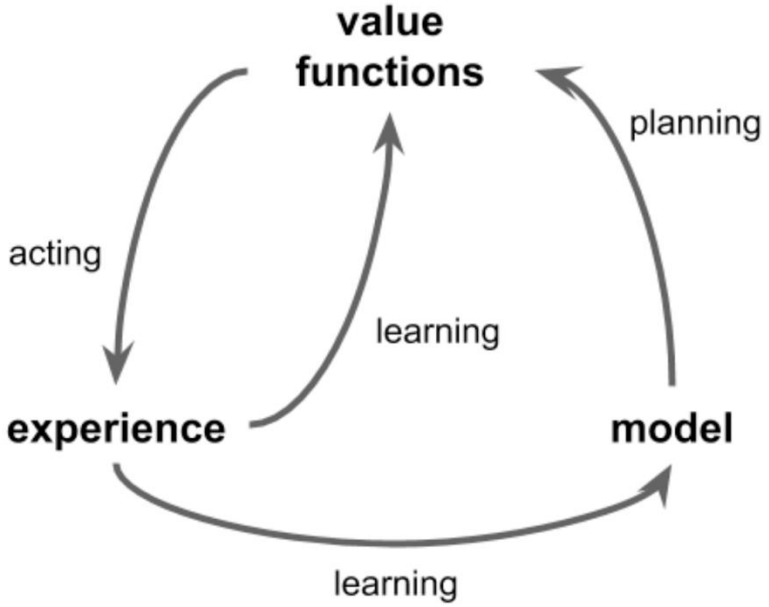

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On-Policy vs. Off-Policy

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  - “Learn on the job”
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  - "Learn on the job"
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- Off-Policy Learning
  - "Look over someone's shoulder"
  - Learn about (target) policy π from experience sampled from (behavior) policy μ

Q-Learning

Consider the off-policy learning setup:

Sample episode from behavior policy $\{S_1, A_1, R_2, \ldots S_{T-1}, A_{T-1}, R_T, S_T\} \sim \mu$

We want to learn the target policy π via learning the Q function

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Here the target policy is the greedy policy: $\pi(a|s) = \begin{cases} 1 & a = \operatorname{argmax}_a Q(s,a) \\ 0 & \text{otherwise} \end{cases}$

Q-Learning

The idea of *Q-Learning* [4] is to learn a Q function that minimizes the Bellman Error. In particular, we can use the fix point iteration to update the Q function iteratively:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \eta \left[\mathcal{R}_{s_t}^{a_t} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

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Exploration-exploitation tradeoff: Q-learning only learns from the state-action pairs it visits. One often needs some strategy to improve the exploration, e.g., ϵ*-greedy policy* [5] (choose optimal action based on Q with probability 1- ϵ and choose a random action with probability ϵ).

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We thus choose the behavior policy to be:

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This ensures that we eventually visit all actions infinitely often!

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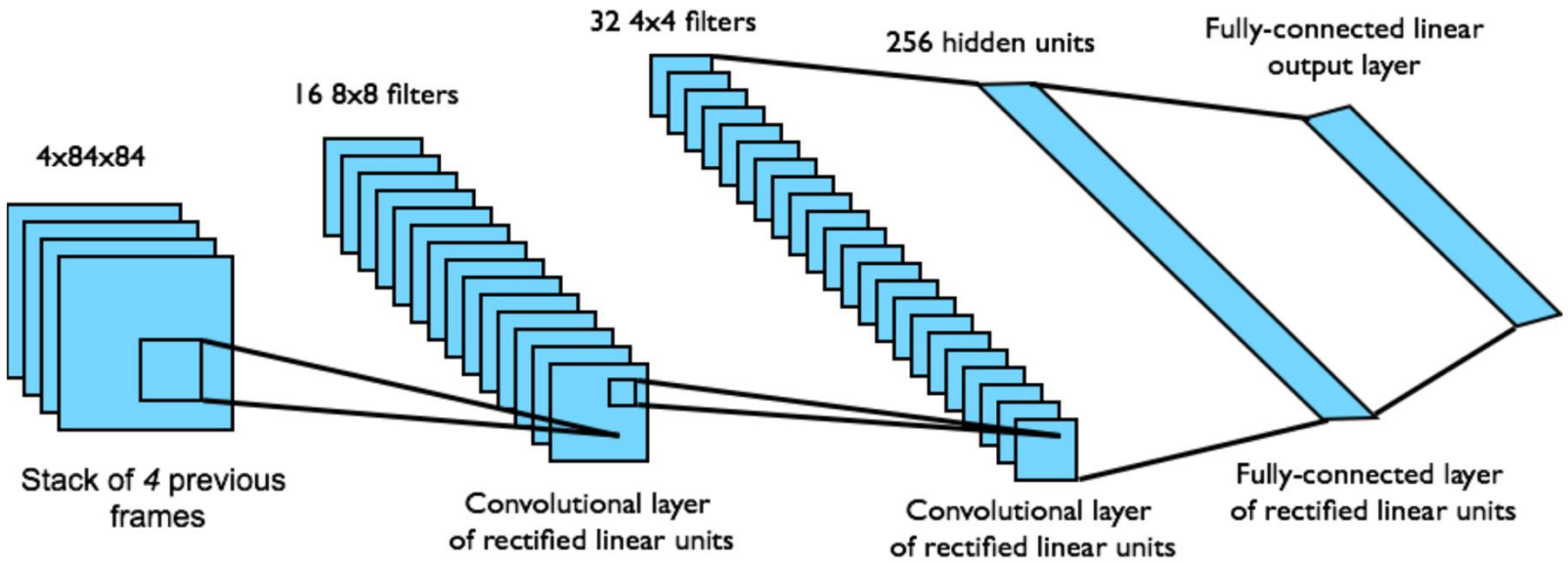
For small state and action spaces, we can represent Q function as a table and learn it. However, for large or infinite spaces, we need to represent it as a parametric function, e.g., a deep neural network!

Deep Q-Learning

Approximating Q function with a neural net is a decades-old idea, but DeepMind got it to work really well on Atari games in 2013 (“deep Q-learning”) [6].



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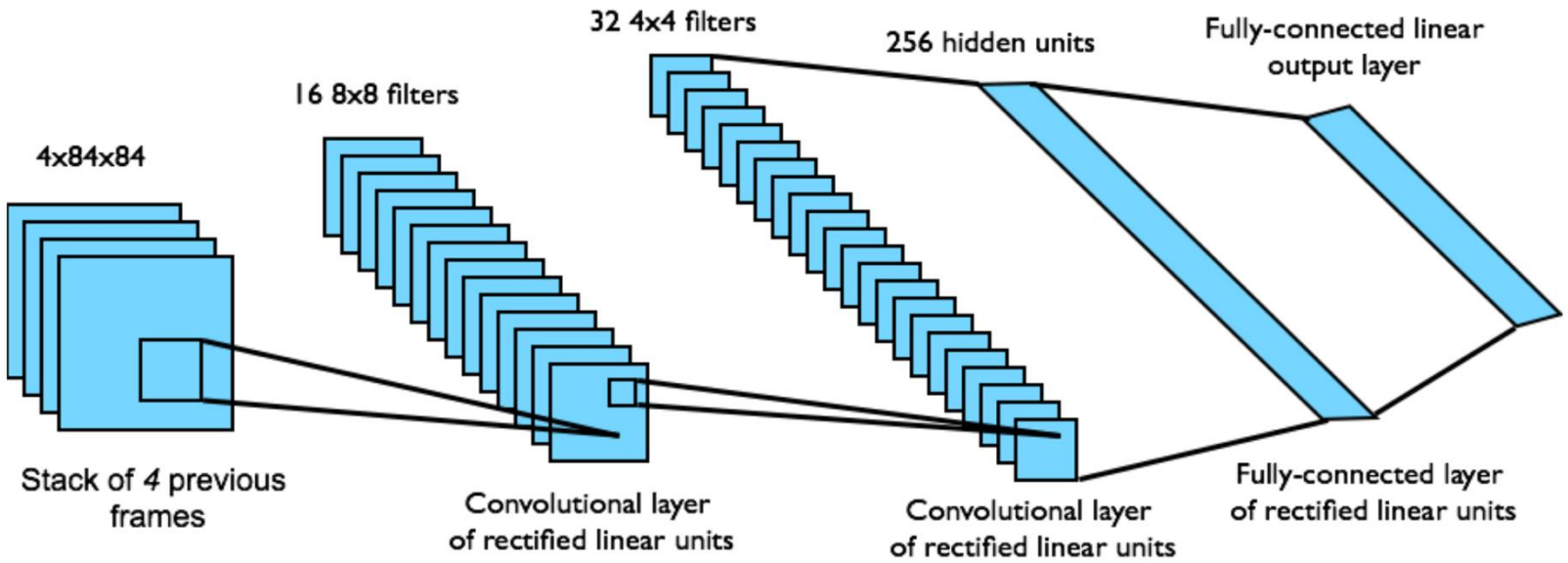

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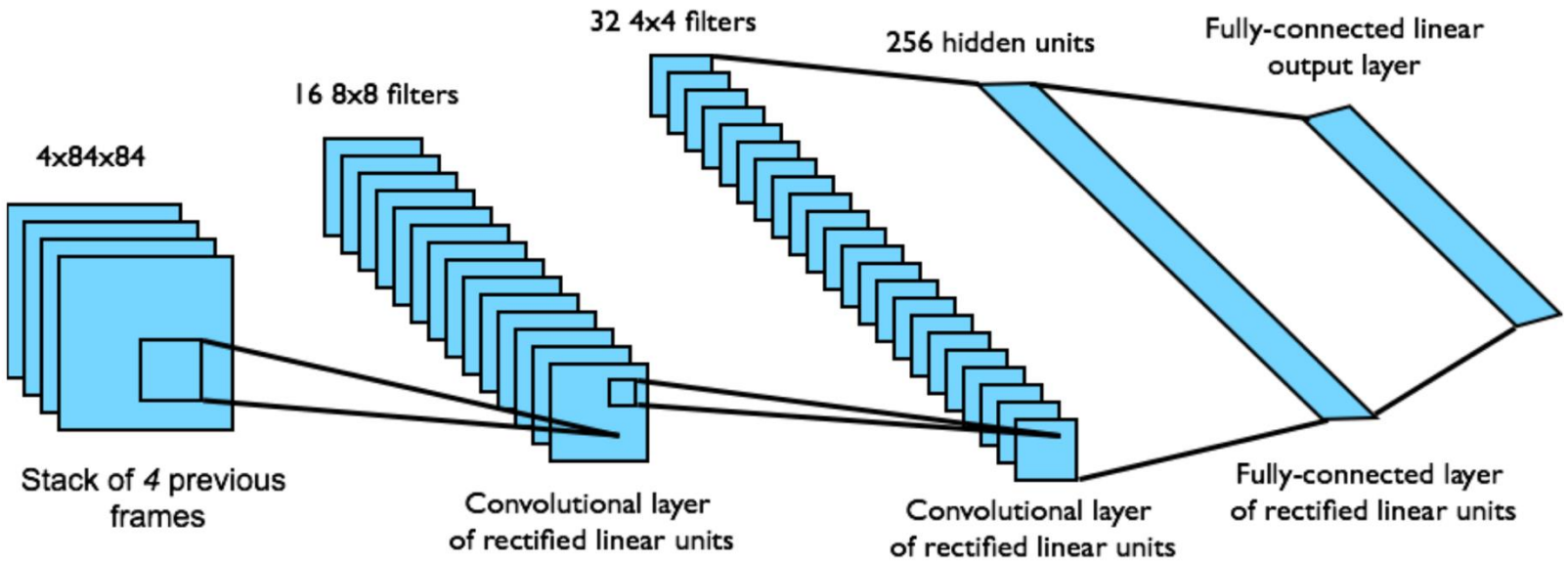

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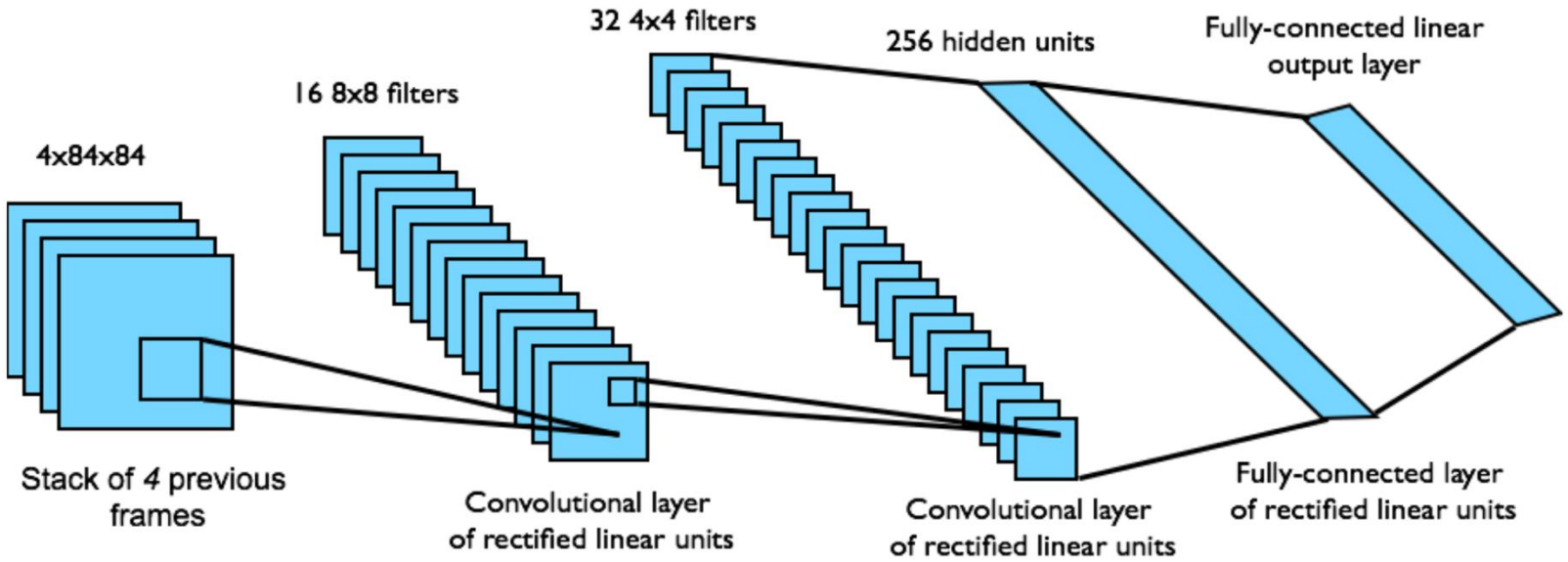
- Compute Q-targets w.r.t. old and fixed parameters $\bar{\theta}$

$$\theta_{t+1} \leftarrow \theta_t + \eta \mathbb{E}_{s_t, a_t, s_{t+1}} \left[\left(\mathcal{R}_{s_t}^{a_t} + \gamma \max_a Q_{\bar{\theta}}(s_{t+1}, a) - Q_{\theta_t}(s_t, a_t) \right) \frac{\partial Q_{\theta_t}(s_t, a_t)}{\partial \theta_t} \right]$$



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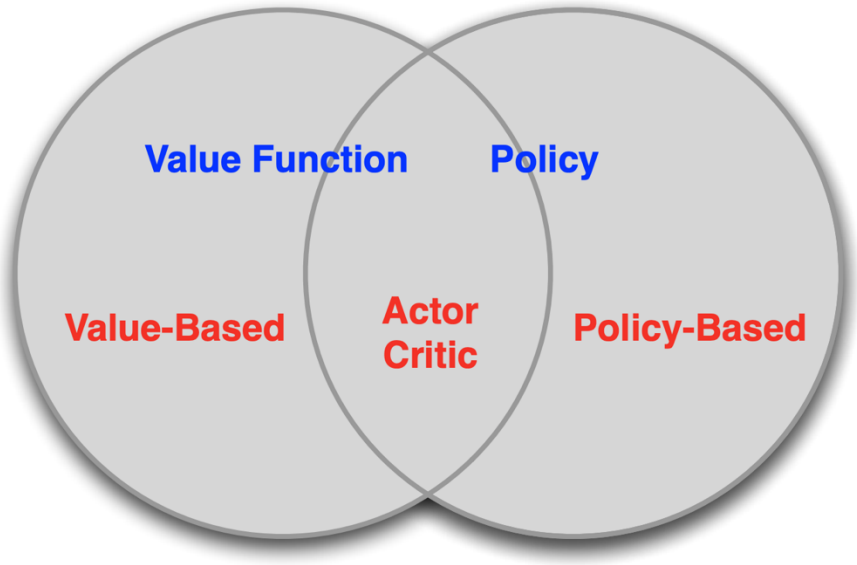

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Disadvantages:

- Often converge to local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

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In policy based methods, we directly parameterize the policy and learn it to maximize some reward.

Given a trajectory $\boldsymbol{\tau}$, let us consider the simple expected reward:

$$J(\theta) = \mathbb{E}_{\boldsymbol{\tau}}\left[R(\boldsymbol{\tau})\right] = \int \mathbb{P}_{\theta}(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

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Log derivative trick:

$$\nabla_\theta J(\theta) = \nabla_\theta \int \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

$$= \int \nabla_\theta \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

$$= \int \mathbb{P}_\theta(\boldsymbol{\tau}) \nabla_\theta \log \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau}) d\boldsymbol{\tau}$$

$$= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau})\right]$$

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Let us substitute $\mathbb{P}_\theta(\boldsymbol{\tau}) = \mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)$ $R(\boldsymbol{\tau}) = \sum_{t=1}^{T} R_t(A_t,S_t)$

We have

$$\begin{aligned}\nabla_\theta J(\theta) &= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log \mathbb{P}_\theta(\boldsymbol{\tau}) R(\boldsymbol{\tau})\right] \\ &= \mathbb{E}_{\boldsymbol{\tau}}\left[\nabla_\theta \log\left(\mathbb{P}_0(S)\prod_{t=1}^{T}\pi_\theta(A_t|S_t)\mathbb{P}(S_{t+1}|S_t,A_t)\right)\left(\sum_{t=1}^{T} R_t(A_t,S_t)\right)\right]\end{aligned}$$

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&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=1}^{T} r_t^{(i)}\right)
\end{aligned}
$$

We use the Monte Carlo Approximation. This is known as REINFORCE algorithm [8]!

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Full Reward

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t^{(i)}|s_t^{(i)})\left(\sum_{t'=t+1}^{T} r_{t'}^{(i)}\right)\right)$$

Reward-to-go: *my current policy can not change past rewards (causality)*!

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This works since

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Since the baseline does not depend on the action, subtracting the baseline still leads to an unbiased estimator. By carefully choosing the baseline, we can reduce the variance!

Advanced Policy Gradient Methods

What is wrong with the Policy Gradient besides high variance?

Similar to other optimization methods, step sizes matter for convergence!

$$\theta' \leftarrow \operatorname{argmax}(\theta' - \theta)^\top \nabla_\theta J(\theta) \quad \text{s.t. } \|\theta' - \theta\|^2 \leq \epsilon$$

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- Natural Policy Gradient [9]: Preconditioning with Fisher-Information Matrix

  $\mathbf{F} = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a|s) \nabla_\theta \log \pi_\theta(a|s)^\top\right]$ $\qquad$ $D_{\mathrm{KL}}(\pi_{\theta'} \| \pi_\theta) \approx (\theta' - \theta)^\top \mathbf{F} (\theta' - \theta)$

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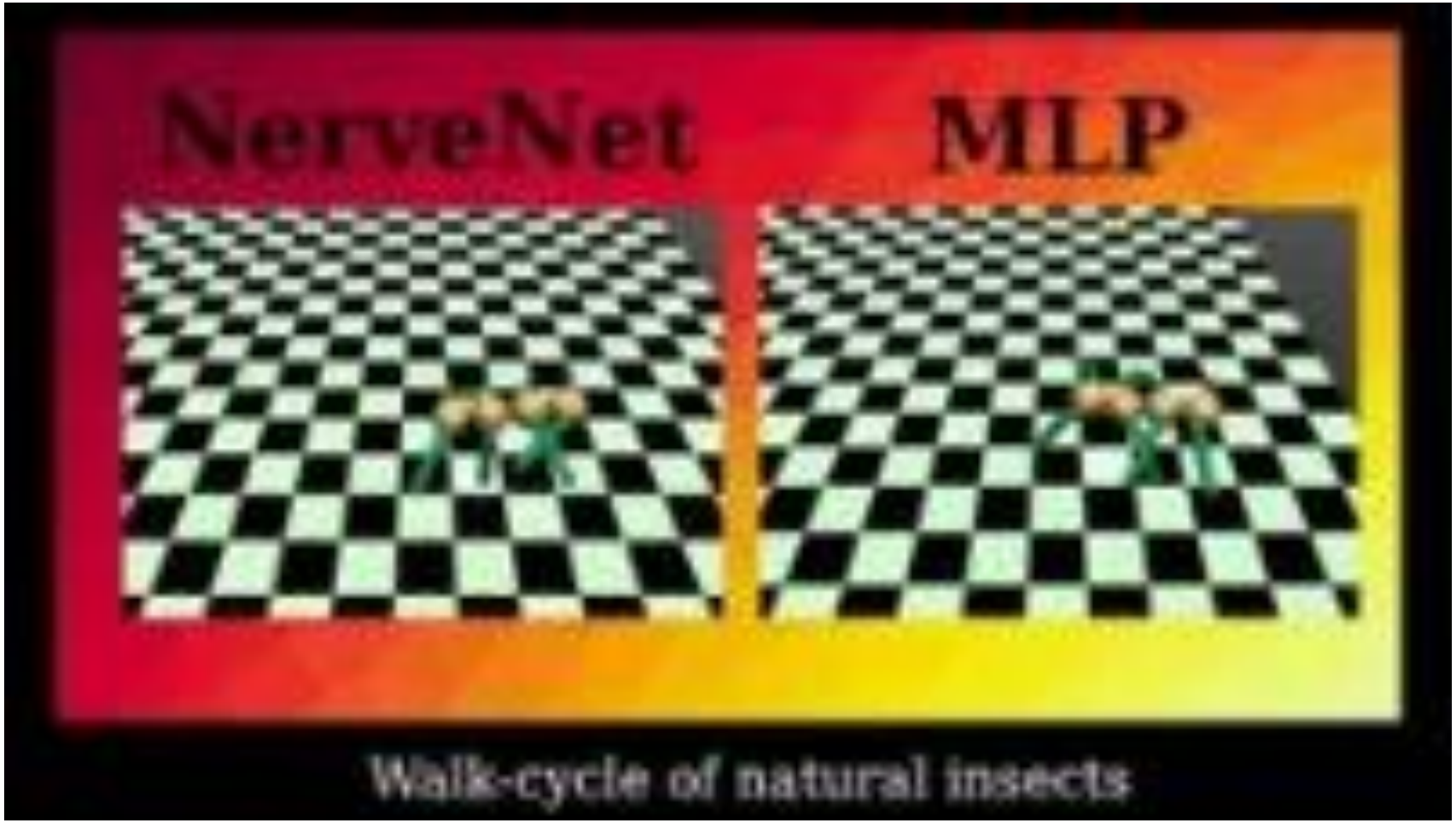
- Proximal Policy Optimization (PPO) [11]: turn above constrained optimization into an unconstrained one (with clipped loss) and use modern 1st-order optimizers like Adam.

Demo

Simulated Continuous Control in Mujoco using PPO [11] and graph neural networks [12]:



Actor-Critic Methods

Recall policy gradient: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)}) \left(\sum_{t'=t+1}^{T} r_{t'}^{(i)} \right) \right)$$

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Q is the true expected reward-to-go and we can use more samples to estimate it!

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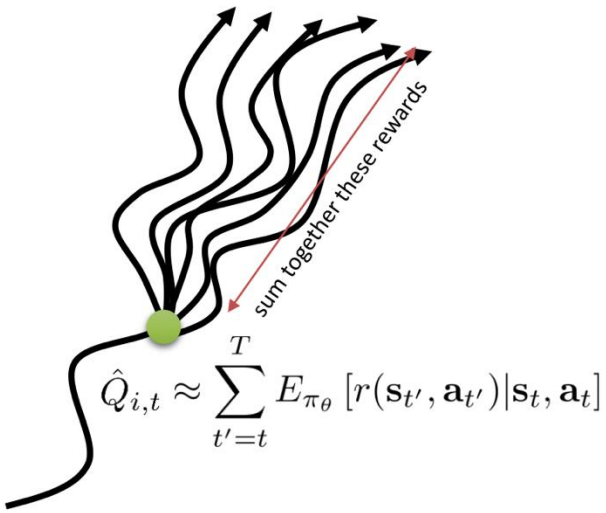

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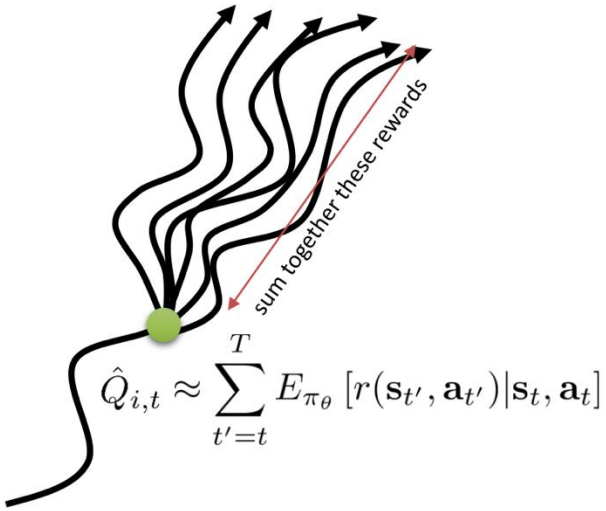

Following the idea of reducing variance via baselines, we introduce the **advantage**:

$$A_\pi(s,a) = Q_\pi(s,a) - V_\pi(s)$$

where the value does not depend on the action, thus serving as the baseline.

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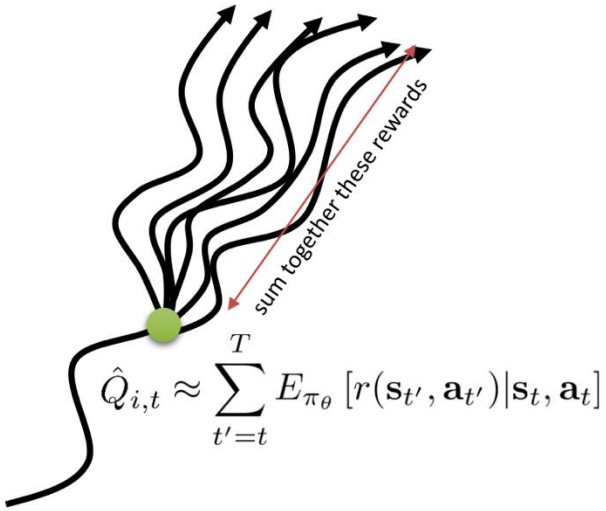

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Actor-Critic Methods

How to estimate the advantage function?

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Temporal Difference (TD) error!

Based on the TD error, we can reduce it to estimating the value function!

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The target can be estimated from rollout:

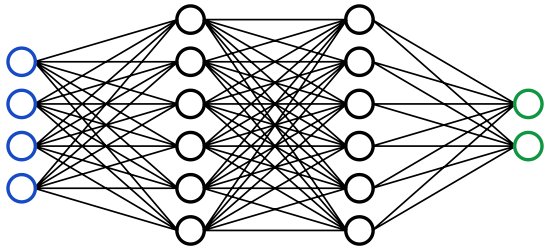
- Vanilla $\bar{V}(s_t^{(i)}) \approx \sum_{t'=t}^{T} r_{t'}^{(i)}$
- Bootstrap $\bar{V}(s_t^{(i)}) \approx r_t^{(i)} + \gamma V_\phi(s_{t+1}^{(i)})$

Actor-Critic Methods

Given the target, we can learn a value network (critic) by minimizing:

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| V_\phi(s_t^{(i)}) - \bar{V}(s_t^{(i)}) \right\|^2$$

Critic = Value $V_\phi(s_t)$



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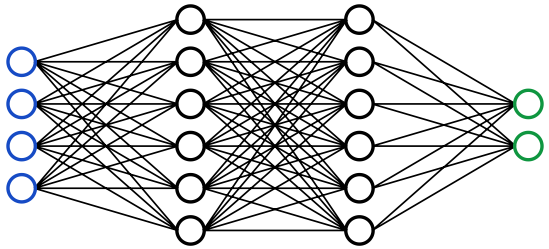

On the other hand, we learn a policy network (actor) by following policy gradient!

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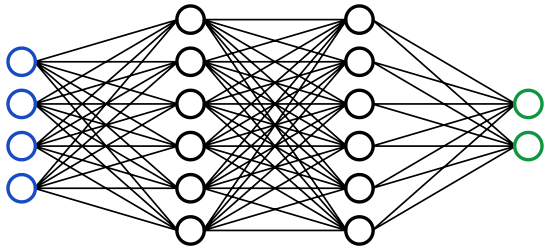

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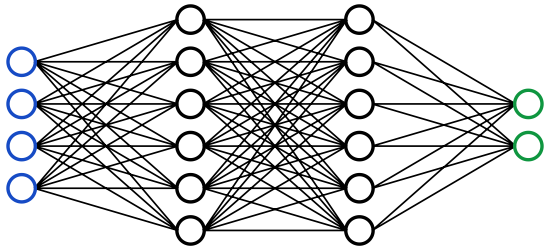

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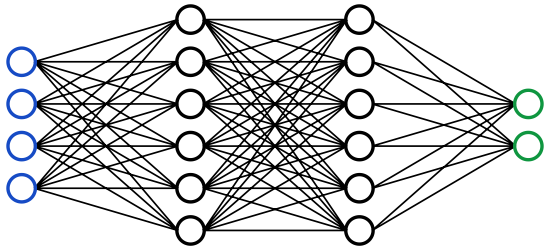

Algorithm 1 (Batch) Actor-Critic Algorithm

Input: Initialize θ and ϕ
Repeat
  Sample $\{\mathbf{a}^{(i)}, \mathbf{s}^{(i)}\} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$
  Update ϕ by minimizing $\frac{1}{2}\sum_i \left\| V_\phi(\mathbf{s}^{(i)}) - \bar{V}(\mathbf{s}^{(i)}) \right\|^2$
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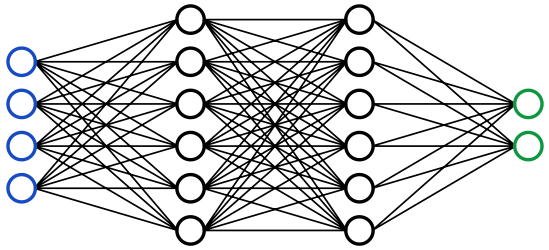

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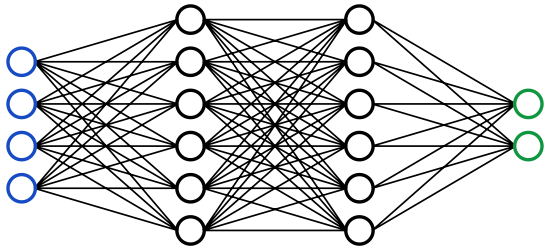

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Episodic trajectories

Outline

- Reinforcement Learning (RL) I
  - Overview & Applications
  - RL Formulation & Taxonomy
  - Markov Decision Process (MDP)
  - Bellman Equations
  - Solving RL problems using the Bellman Equations
- Reinforcement Learning (RL) II
  - Model-Free RL: (Deep) Q-learning
  - Model-Free RL: Policy Gradient & Actor-Critic
  - **Model-Based RL: Dyna-Q**

Model-Based RL

Advantages:

- Can efficiently learn models via supervised learning
- Can reason about model uncertainty

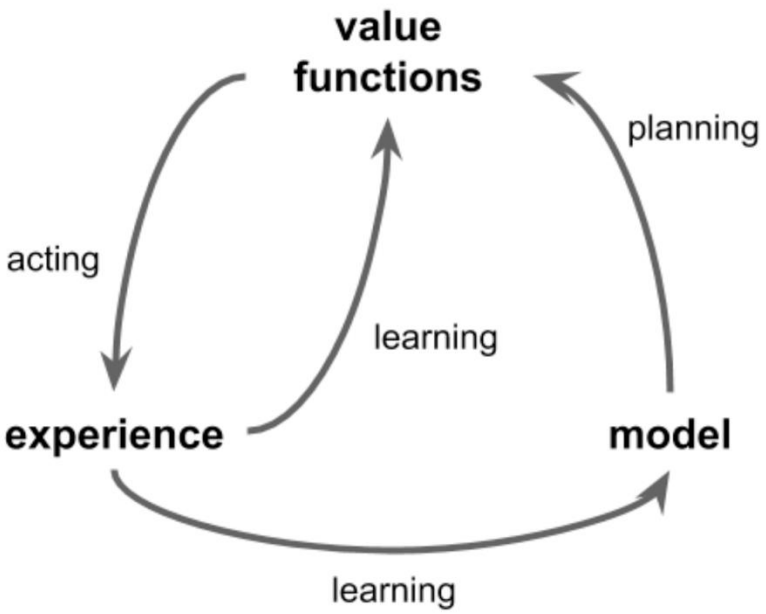

Image Credit: [3]

Model-Based RL

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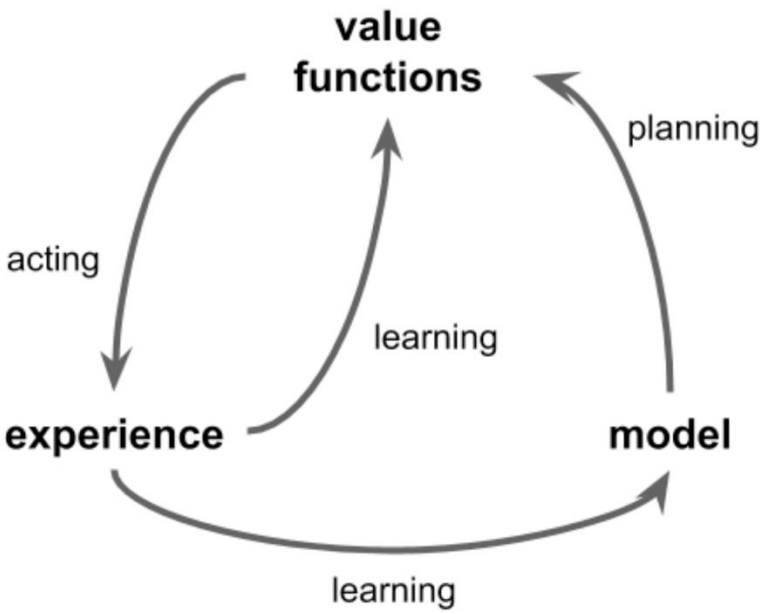
Disadvantages:

- First learn a model and then do planning
  → Two sources of approximation error
- Sometimes learning a model is harder than solving the tasks at hand



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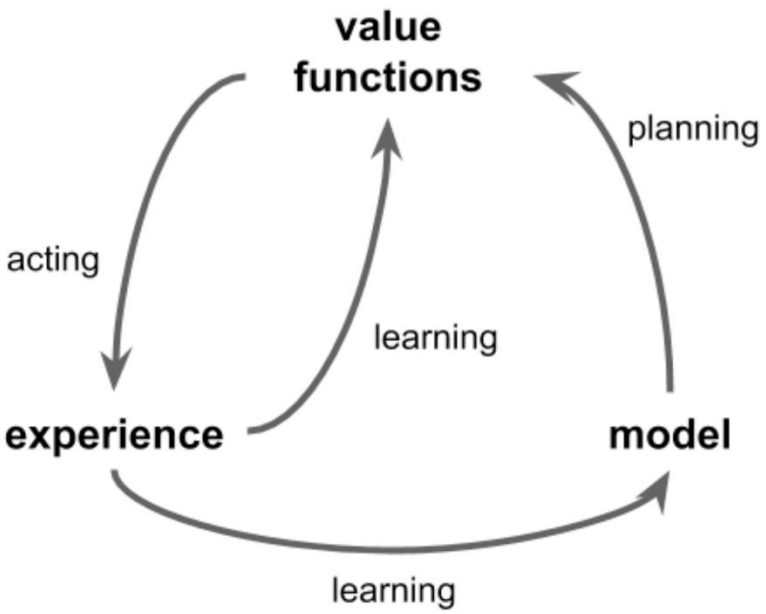
Model-Based RL

Advantages:

- Can efficiently learn models via supervised learning
- Can reason about model uncertainty

Disadvantages:

- First learn a model and then do planning
  → Two sources of approximation error
- Sometimes learning a model is harder than solving the tasks at hand



Recall what a model is, e.g., an MDP that represents how state transits and what the next reward should be!

Image Credit: [3]

Dyna-Q

Dyna-Q [14] combines model-free and model-based RL:

- Learn a model from real experience
- Learn and plan value function (and/or policy) from real and simulated experience

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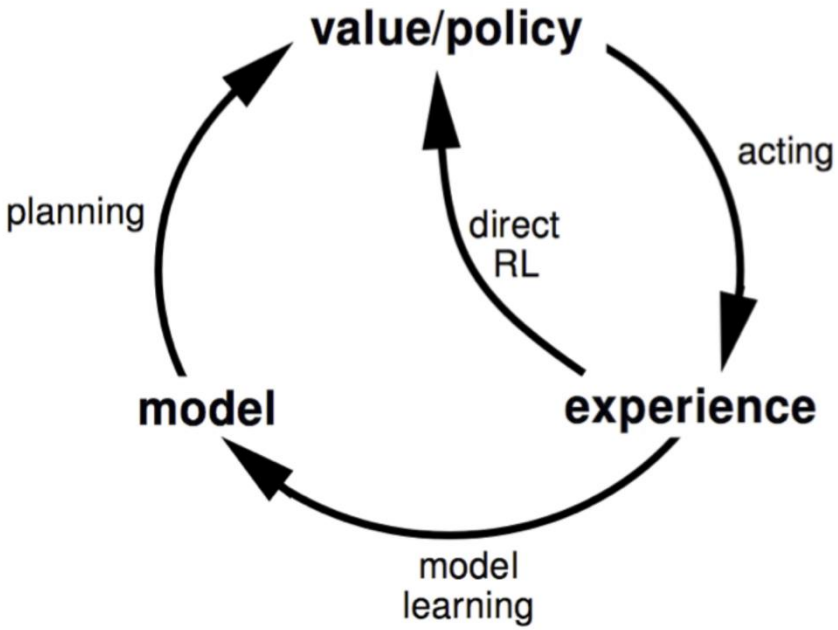

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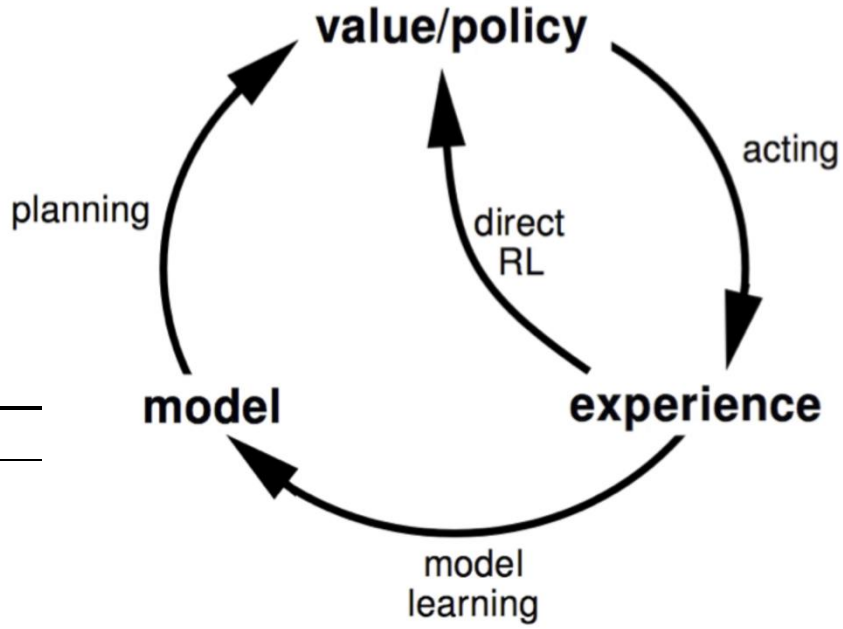

Algorithm 2 Dyna-Q Algorithm

Input: Initialize $Q(s,a)$ and Model(s,a) for all s and a
Do forever
  $s \leftarrow$ current (nonterminal) state
  $a \leftarrow \epsilon$-greedy(s,Q)
  Execute action a, observe reward r and state s'
  $Q(s,a) \leftarrow Q(s,a) + \eta\left[r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right]$
  Model$(s,a) \leftarrow r, s'$
  Repeat n times
    $s \leftarrow$ random previously observed state
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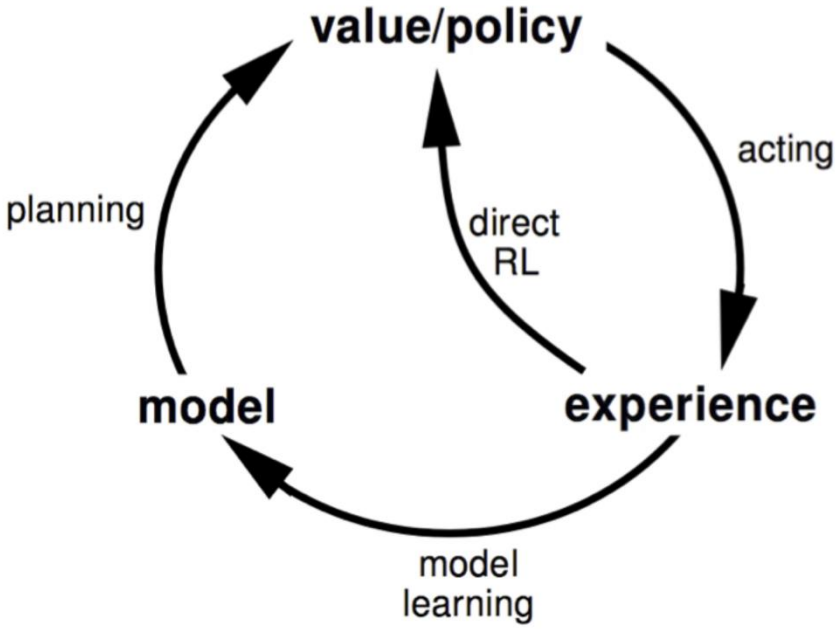

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Supervised learning from real experience!

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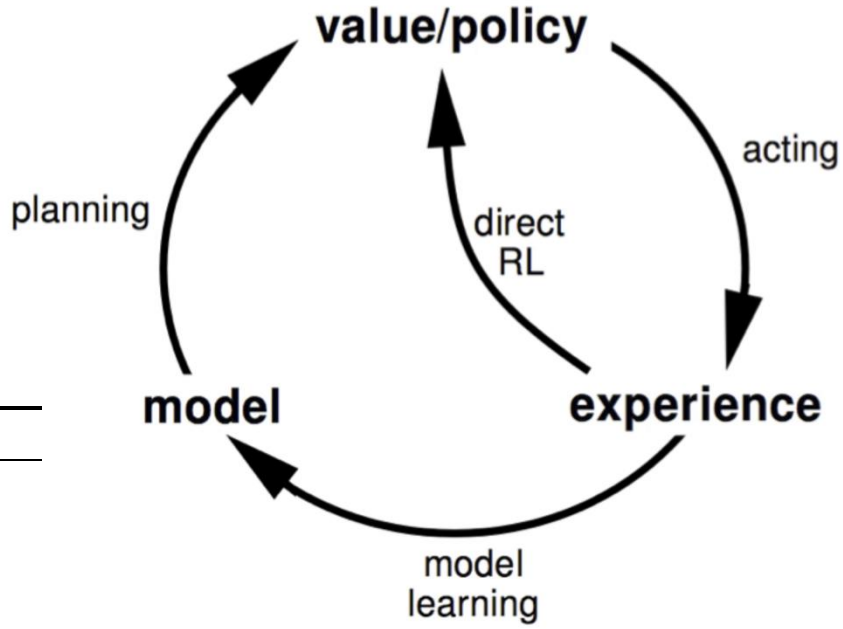

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Supervised learning from real experience!

Learning from simulated experience (imagination)!

Image Credit: [15]

References

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[2] https://www.davidsilver.uk/wp-content/uploads/2020/03/pg.pdf

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[15] https://www.davidsilver.uk/wp-content/uploads/2020/03/dyna.pdf

Questions?