CPEN 455: Deep Learning

Lecture 3: Multilayer Perceptron

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Winter, Term 2, 2024

Outline

- Multilayer Perceptron (MLP)
  - Linear Layer
  - Nonlinear Activation
  - Batch Normalization
  - Dropout
  - Build a Deep MLP

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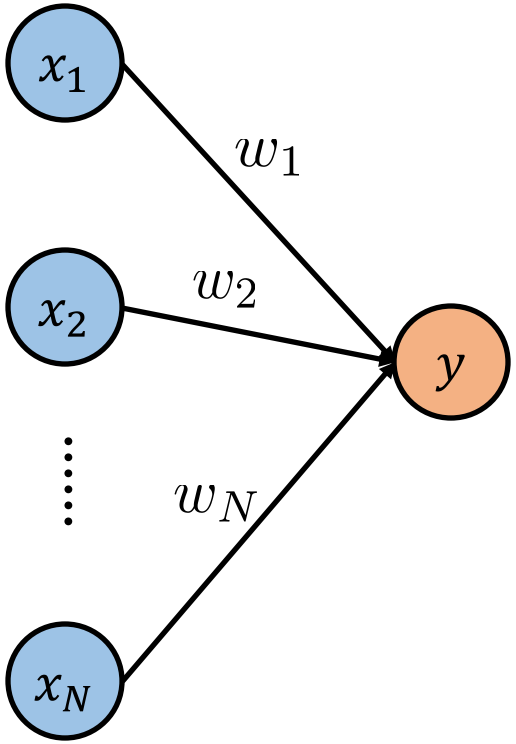
Linear Layer

Consider the following linear layer of a single output unit:



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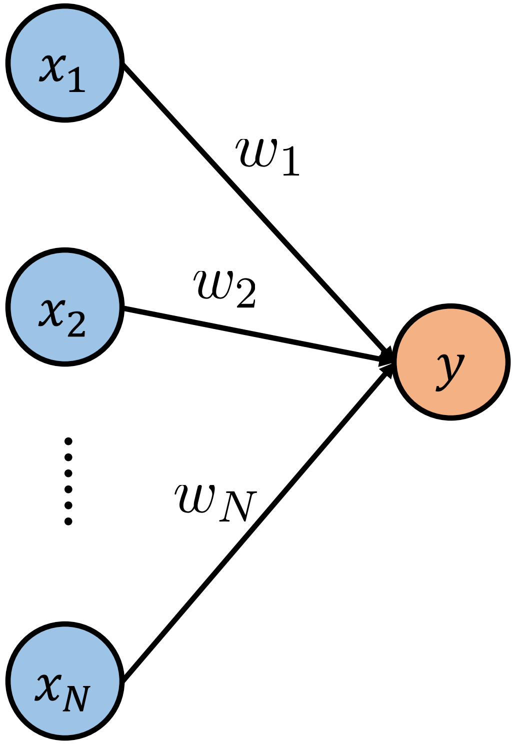

$$y = \sum_{n=1}^{N} w_n x_n$$

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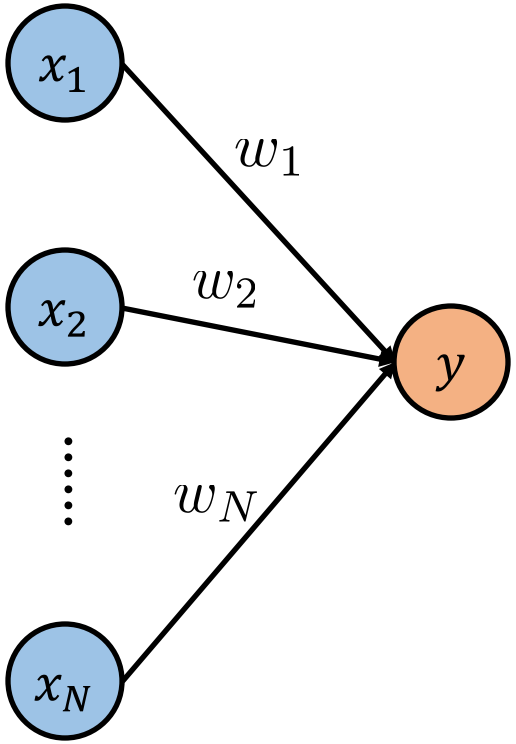

$$y = \sum_{n=1}^{N} w_n x_n$$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

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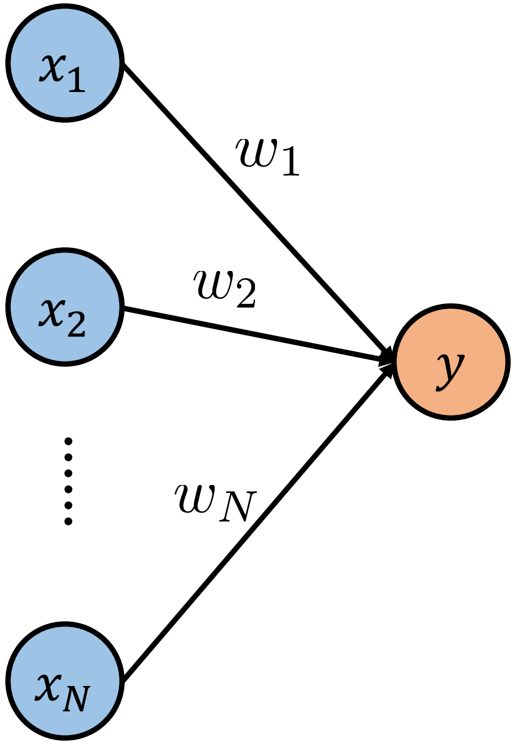


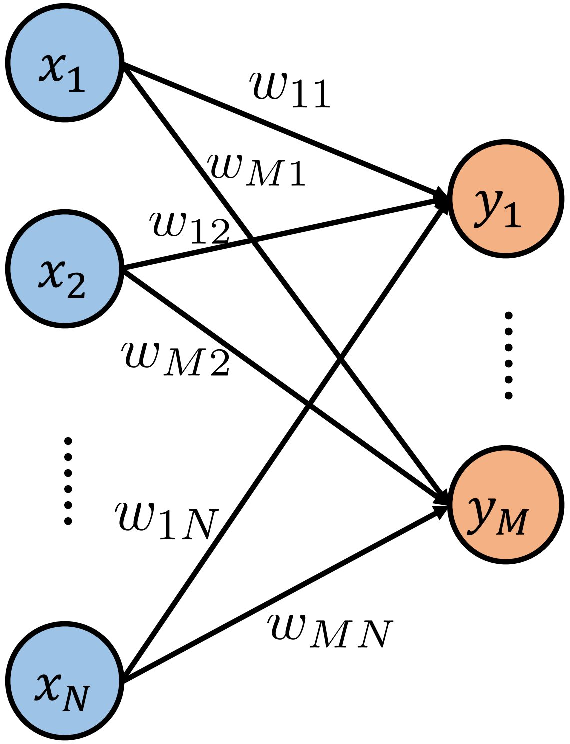
$$y = \sum_{n=1}^{N} w_n x_n = \mathbf{w}^\top \mathbf{x}$$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

Linear Layer

What if we have multiple output units?

$$y_m = \sum_{n=1}^{N} w_{mn} x_n$$



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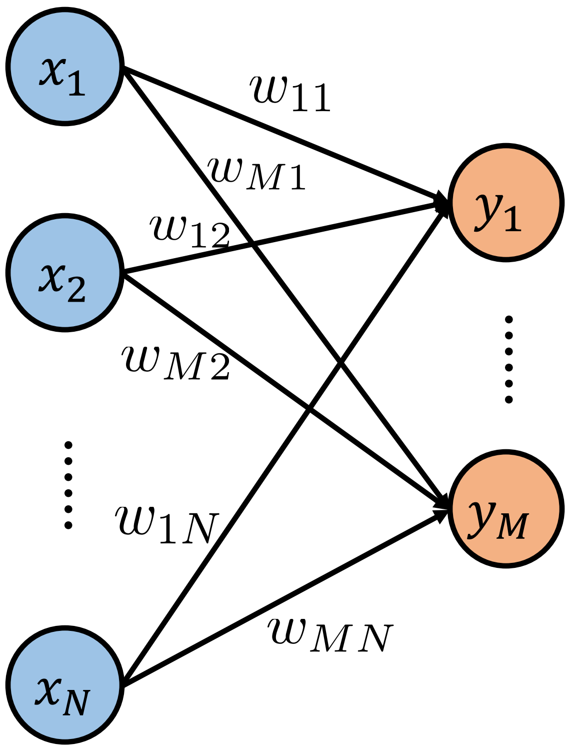

$$y_m = \sum_{n=1}^{N} w_{mn} x_n$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \qquad W = \begin{bmatrix} w_{11}, & w_{12}, & \cdots, & w_{1N} \\ w_{21}, & w_{22}, & \cdots, & w_{2N} \\ \vdots, & \vdots, & \vdots, & \vdots \\ w_{M1}, & w_{M2}, & \cdots, & w_{MN} \end{bmatrix}$$

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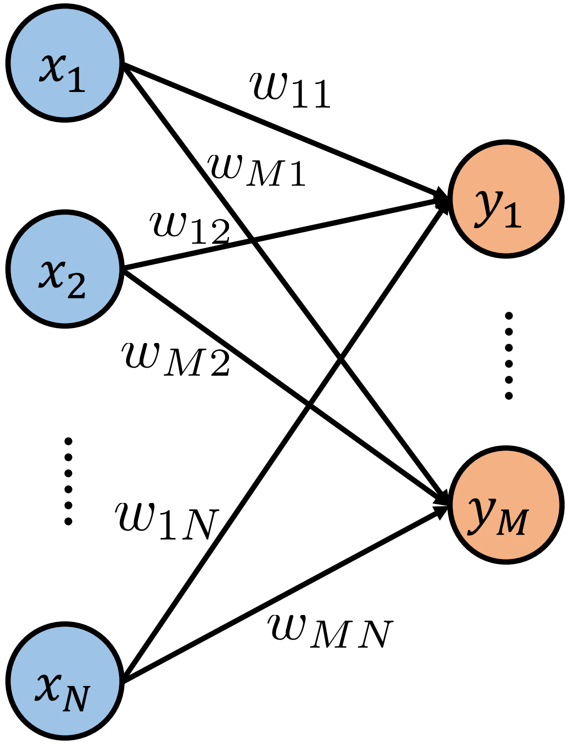

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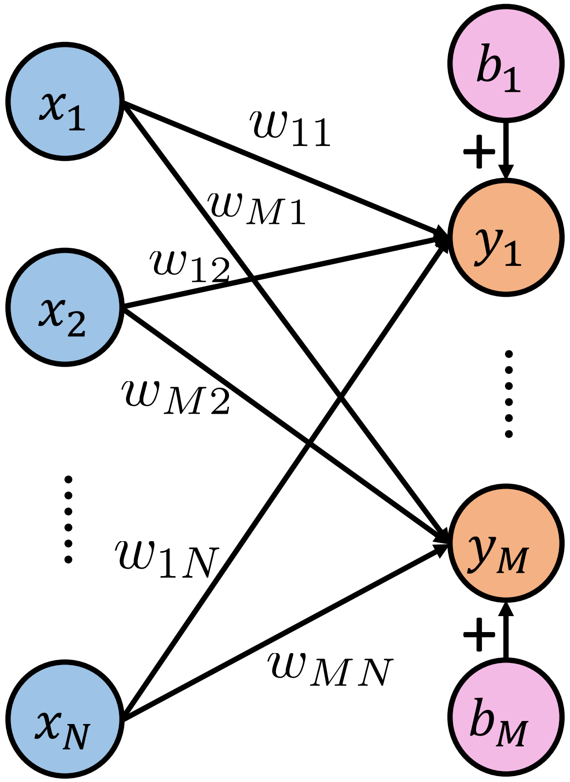
$$\mathbf{y} = W\mathbf{x}$$

Linear Layer

What if we have multiple output units? What about bias?



$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}$$

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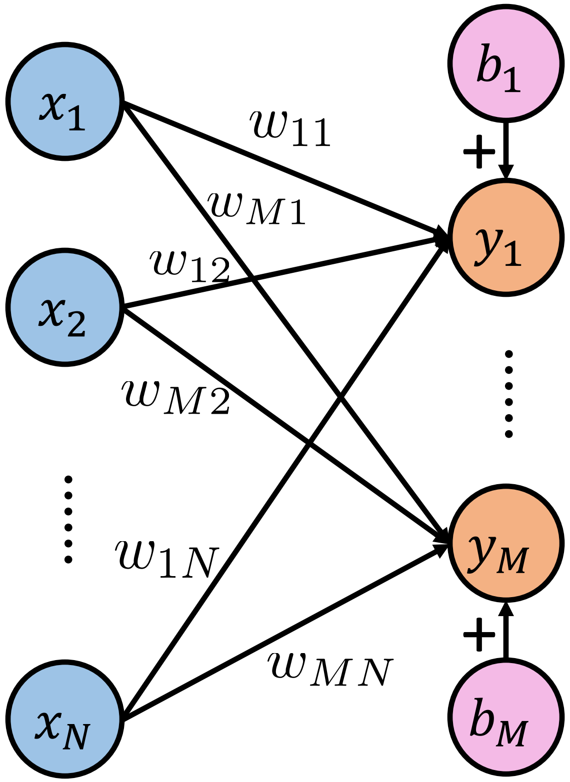


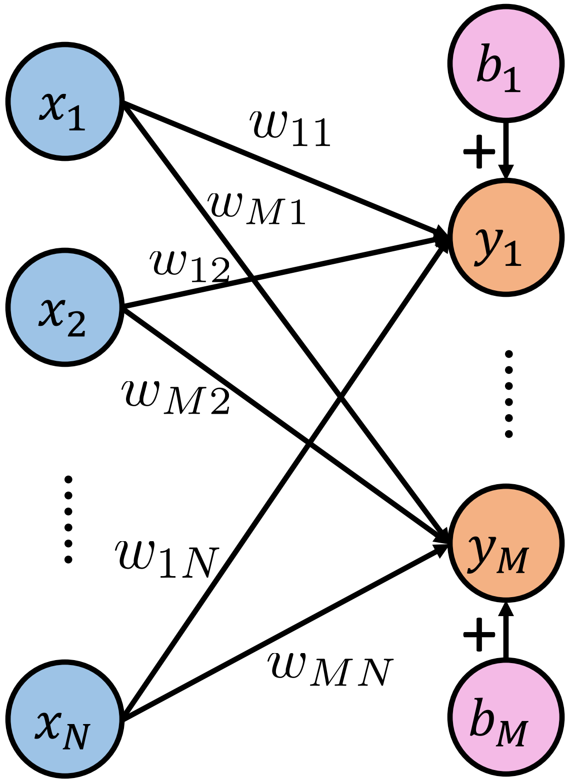
$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}$$

$$\mathbf{y} = W\mathbf{x} + \mathbf{b}$$

Linear Layer

What if we have multiple output units? What about bias?

We can compactly rewrite it via homogeneous coordinates.



$$\tilde{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ 1 \end{bmatrix}$$

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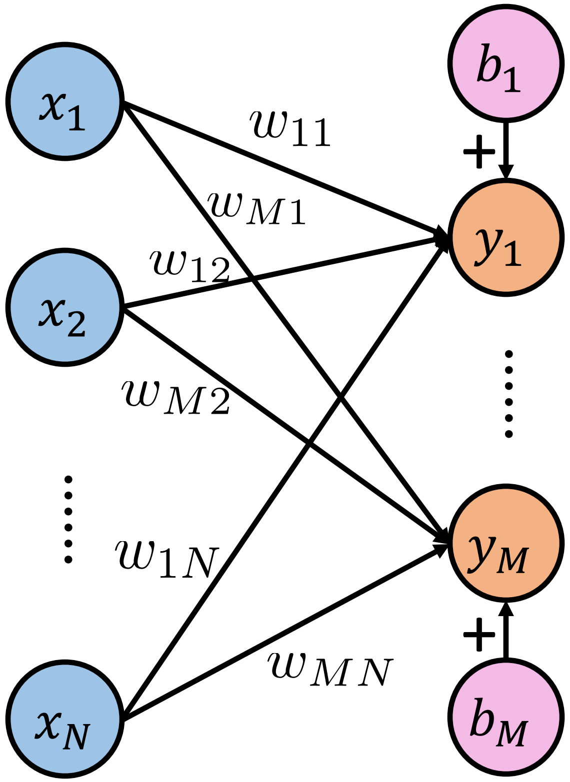

$$\tilde{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ 1 \end{bmatrix} \quad \tilde{W} = \begin{bmatrix} w_{11}, & w_{12}, & \cdots, & w_{1N} & b_1 \\ w_{21}, & w_{22}, & \cdots, & w_{2N} & b_2 \\ \vdots, & \vdots, & \vdots, & \vdots & \vdots \\ w_{M1}, & w_{M2}, & \cdots, & w_{MN} & b_M \end{bmatrix}$$

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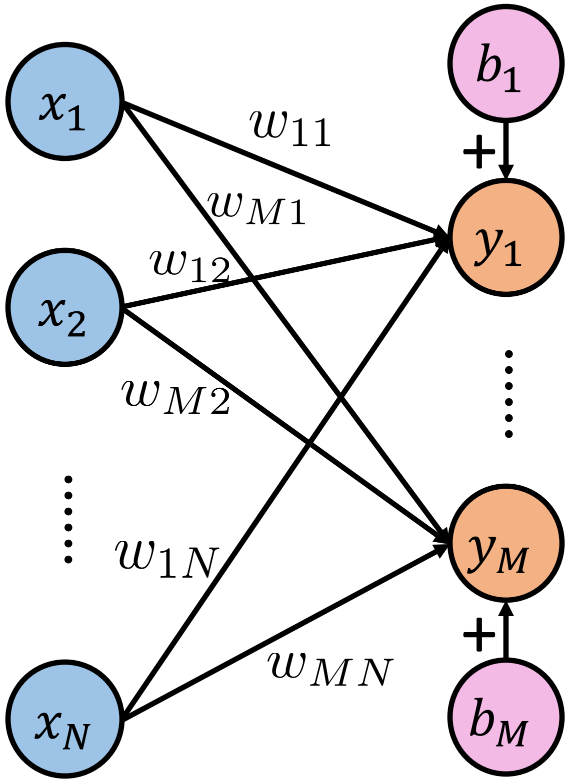

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$$\mathbf{y} = W\mathbf{x} + \mathbf{b} = \tilde{W}\tilde{\mathbf{x}}$$

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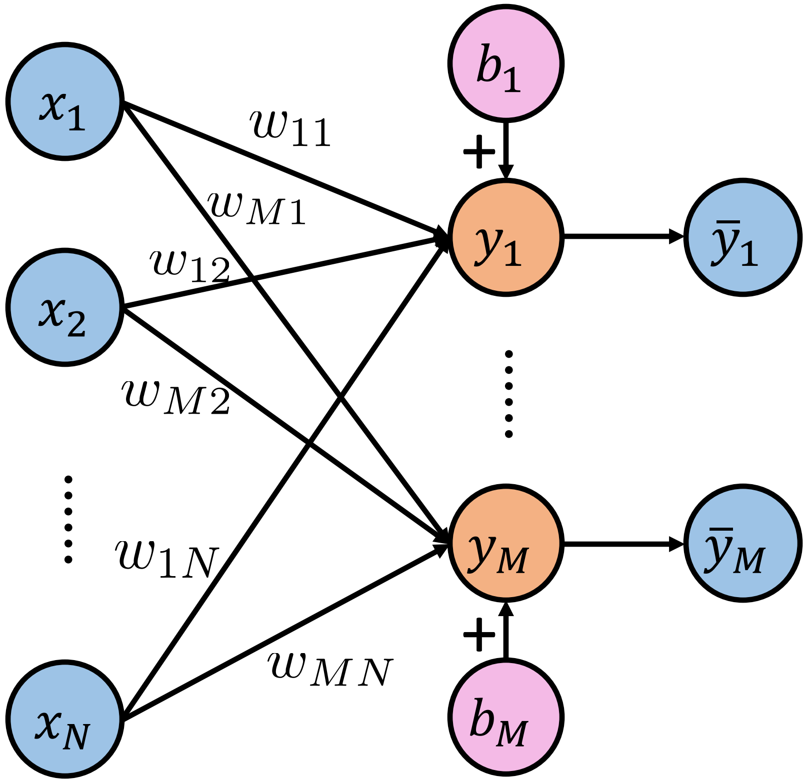
Nonlinear Activation

To make neural networks become nonlinear models, we often apply **element-wise** nonlinear activation functions.



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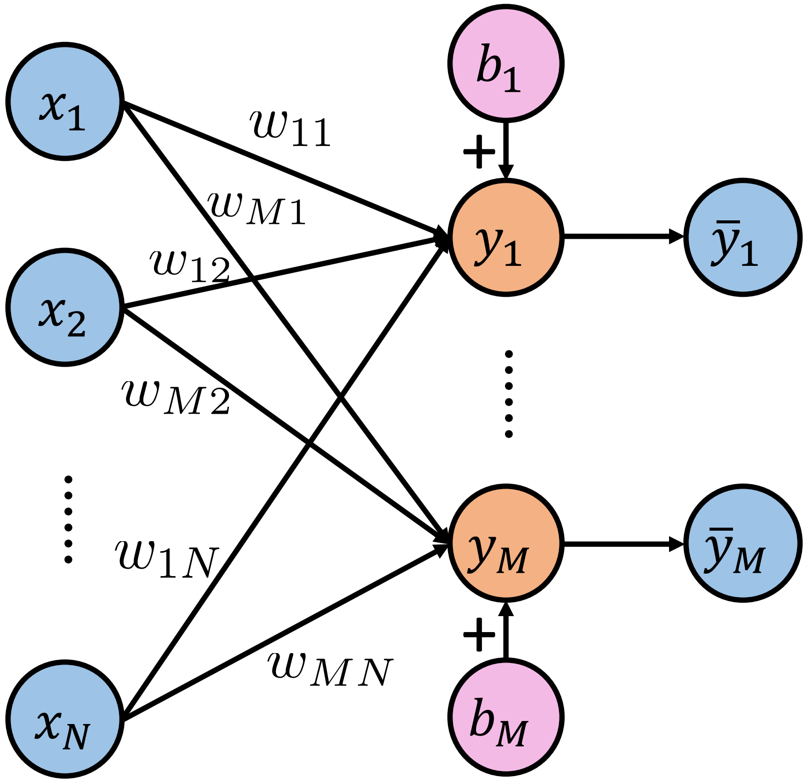


$$\bar{y}_i = f(y_i)$$

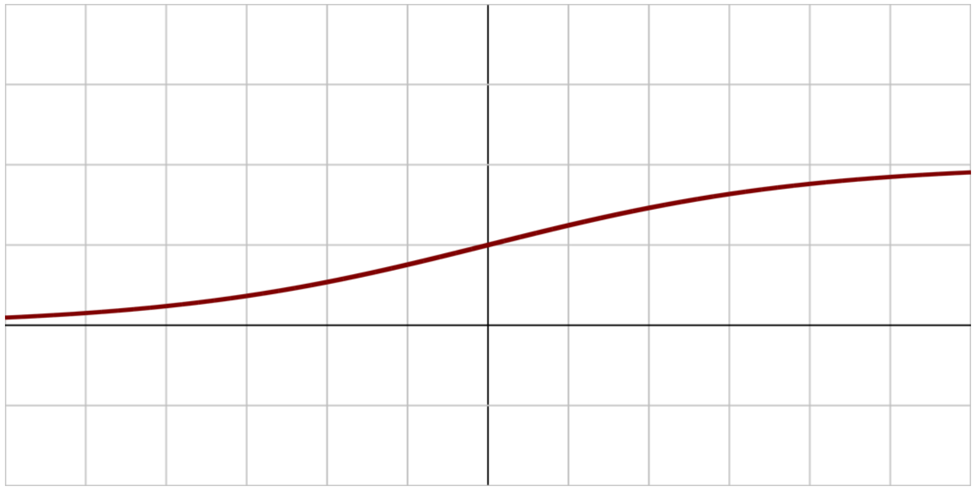
$$\bar{\mathbf{y}} = f(\mathbf{y})$$

People would clarify it if the nonlinear activation is not element-wise.

Nonlinear Activation

To make neural networks become nonlinear models, we often apply **element-wise** nonlinear activation functions.

- **Sigmoid** $$f(x) = \frac{1}{1 + e^{-x}}$$

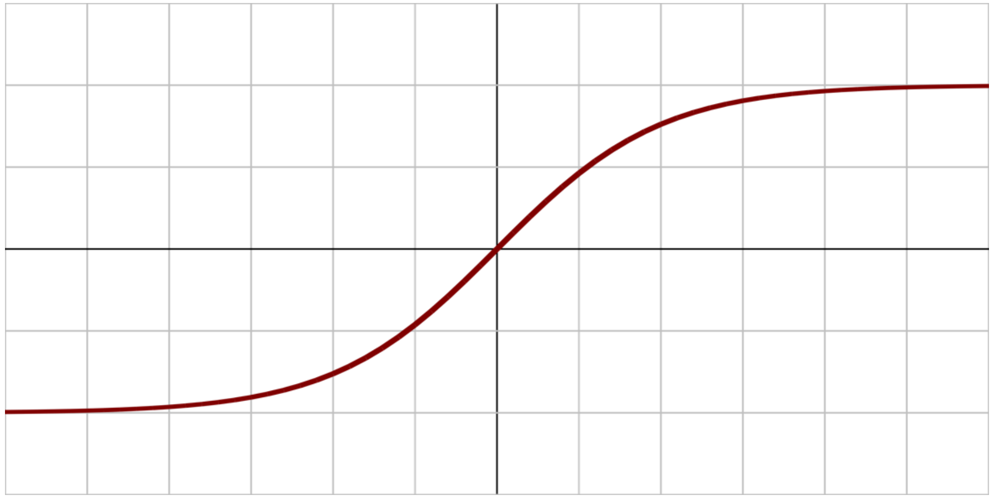


Nonlinear Activation

To make neural networks become nonlinear models, we often apply **element-wise** nonlinear activation functions.

- Sigmoid
- **Tanh**

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



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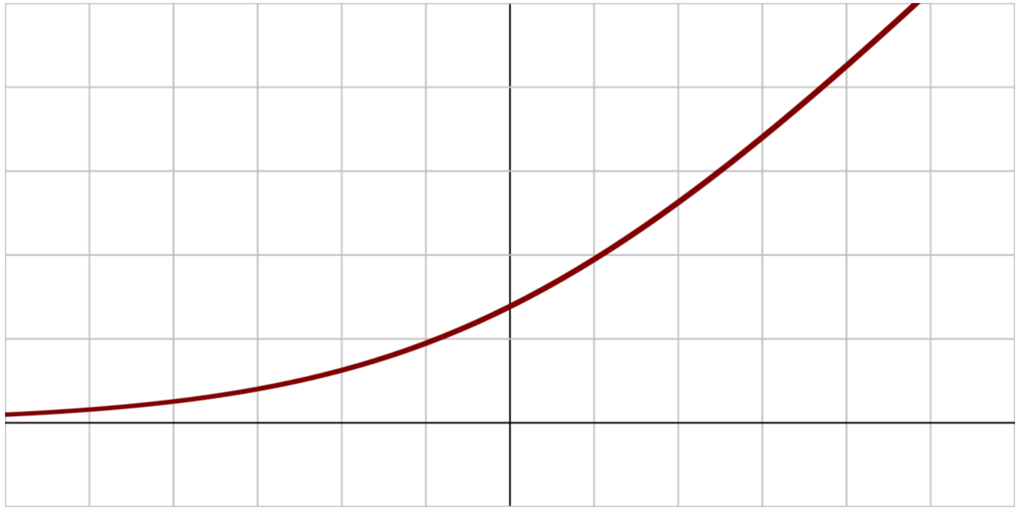
- Sigmoid
- Tanh
- **Softplus [1]**

$$f(x) = \ln(1 + e^x)$$



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- Sigmoid
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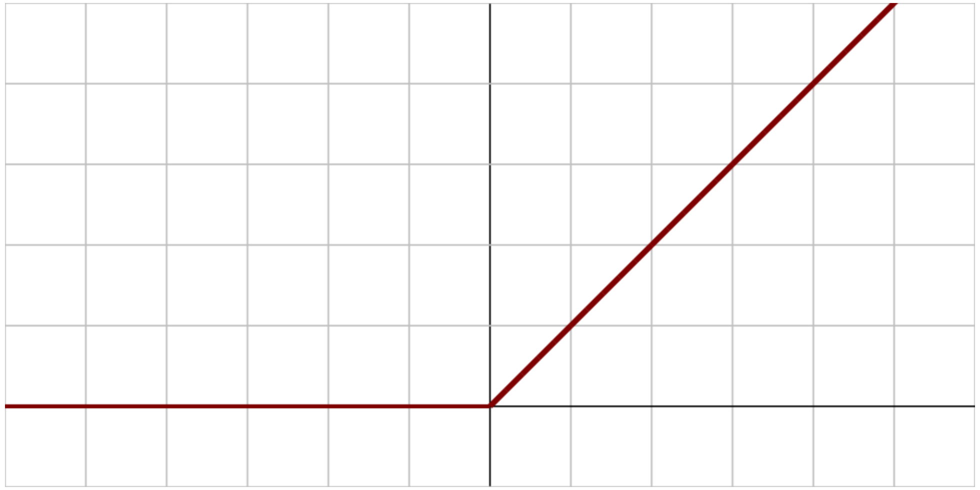
- Softplus [1]
- **Rectified Linear Units (ReLU) [2]**

$$f(x) = \max(x, 0)$$



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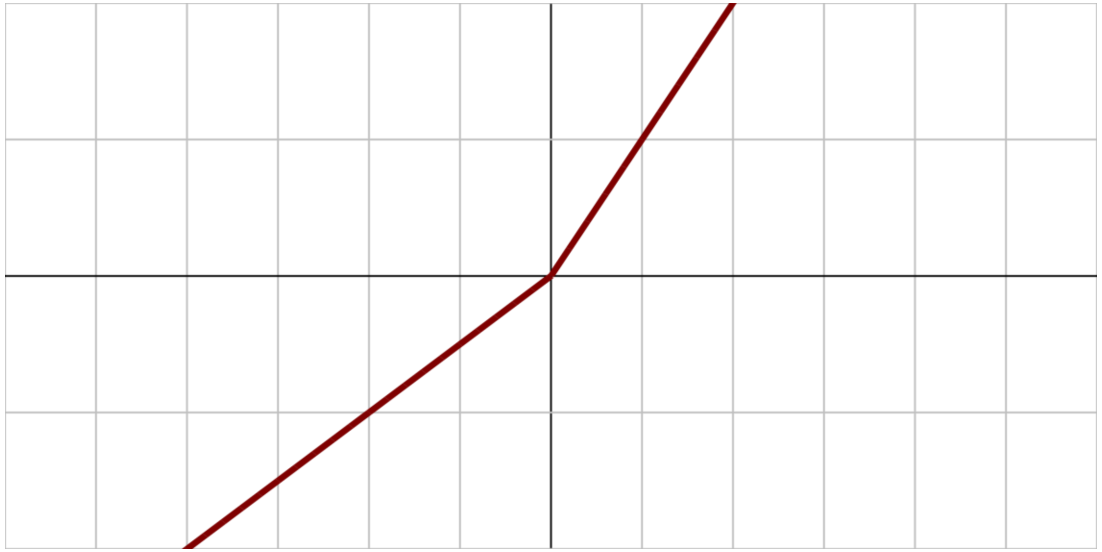
- **Parametric rectified linear unit (PReLU) [3]**

$$f(x) = \begin{cases} \alpha x & \text{if } x < 0 \\ x & \text{otherwise} \end{cases}$$



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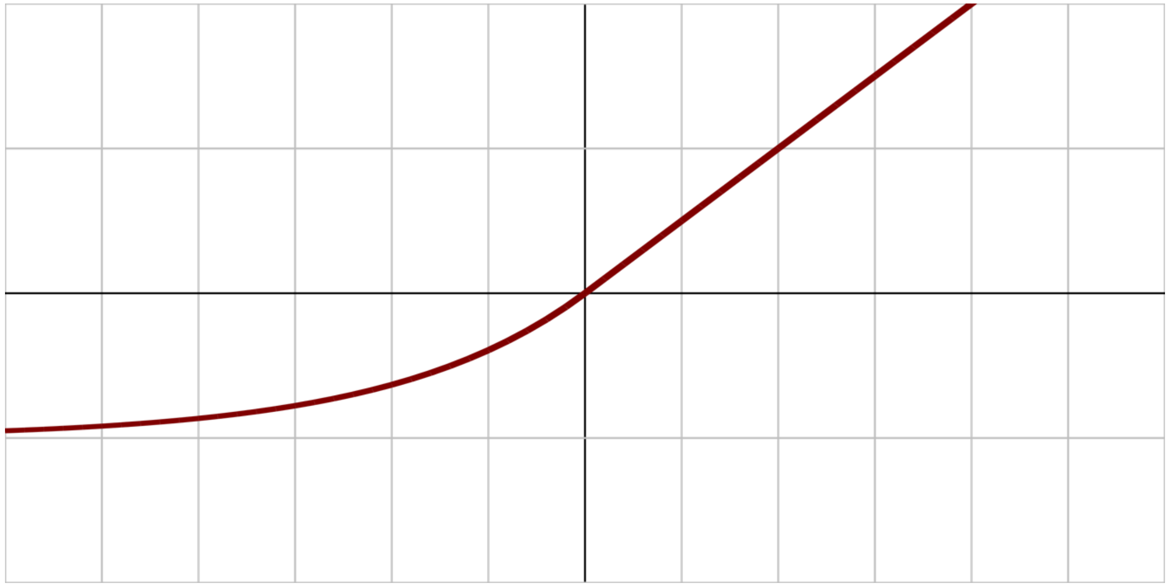
$$f(x) = \begin{cases} \alpha(e^x - 1) & \text{if } x < 0 \\ x & \text{otherwise} \end{cases}$$

- Sigmoid
- Tanh
- Softplus [1]
- Rectified Linear Units (ReLU) [2]
- Parametric rectified linear unit (PReLU) [3]
- **Exponential linear unit (ELU) [4]**



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$$f(\mathbf{x}) = \left[\frac{\exp(\mathbf{x}_1)}{\sum_{k=1}^{K}\exp(\mathbf{x}_k)}, \ldots, \frac{\exp(\mathbf{x}_K)}{\sum_{k=1}^{K}\exp(\mathbf{x}_k)}\right]$$

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- Cummax [6]

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Softmax followed by cumulative sum

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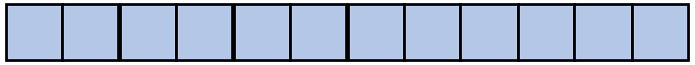
Normalizing pre-activations helps avoid the saturation region of nonlinear activation function

Batch Normalization

Suppose we have a batch of activations $X \in \mathbb{R}^{B \times D}$

vectorize



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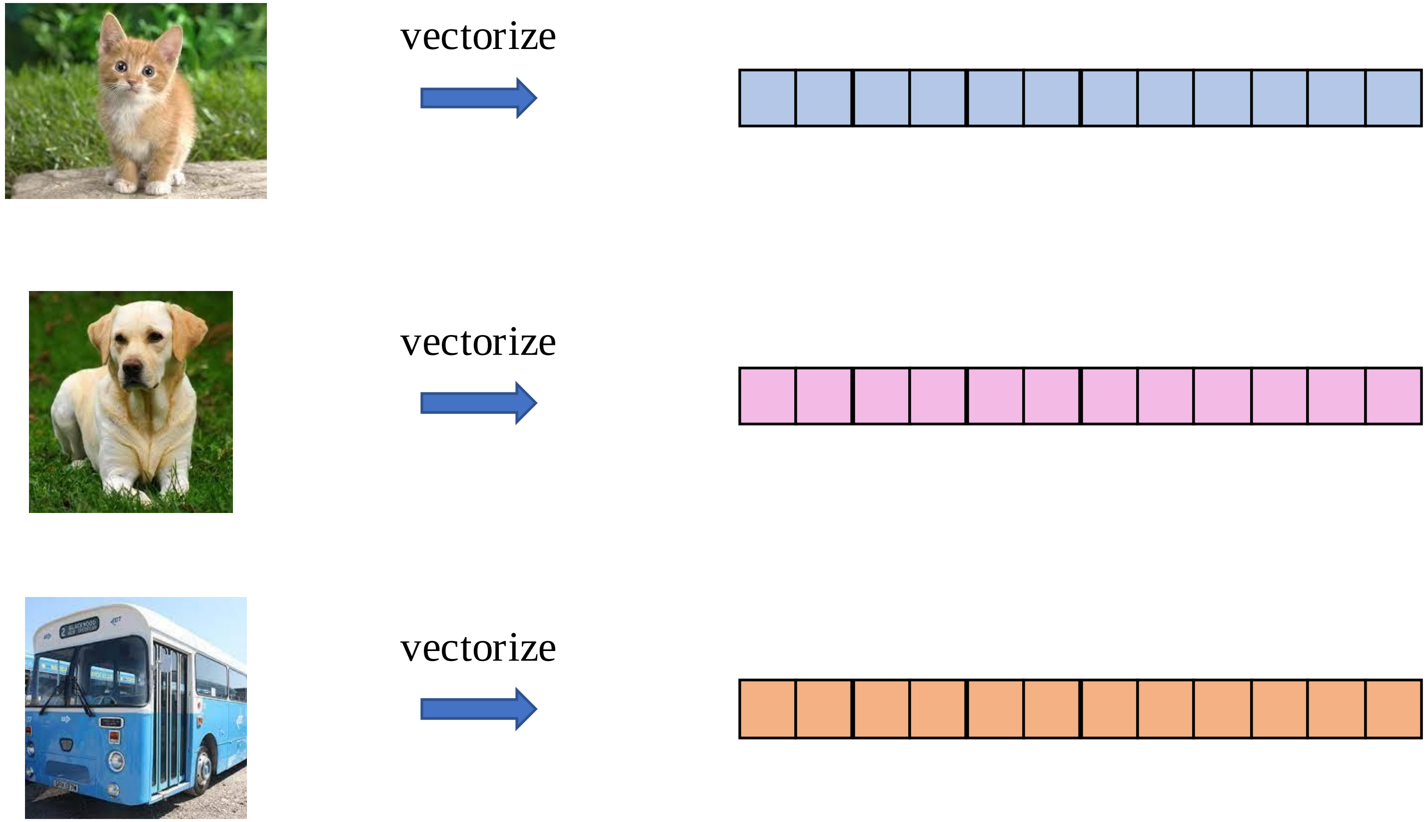

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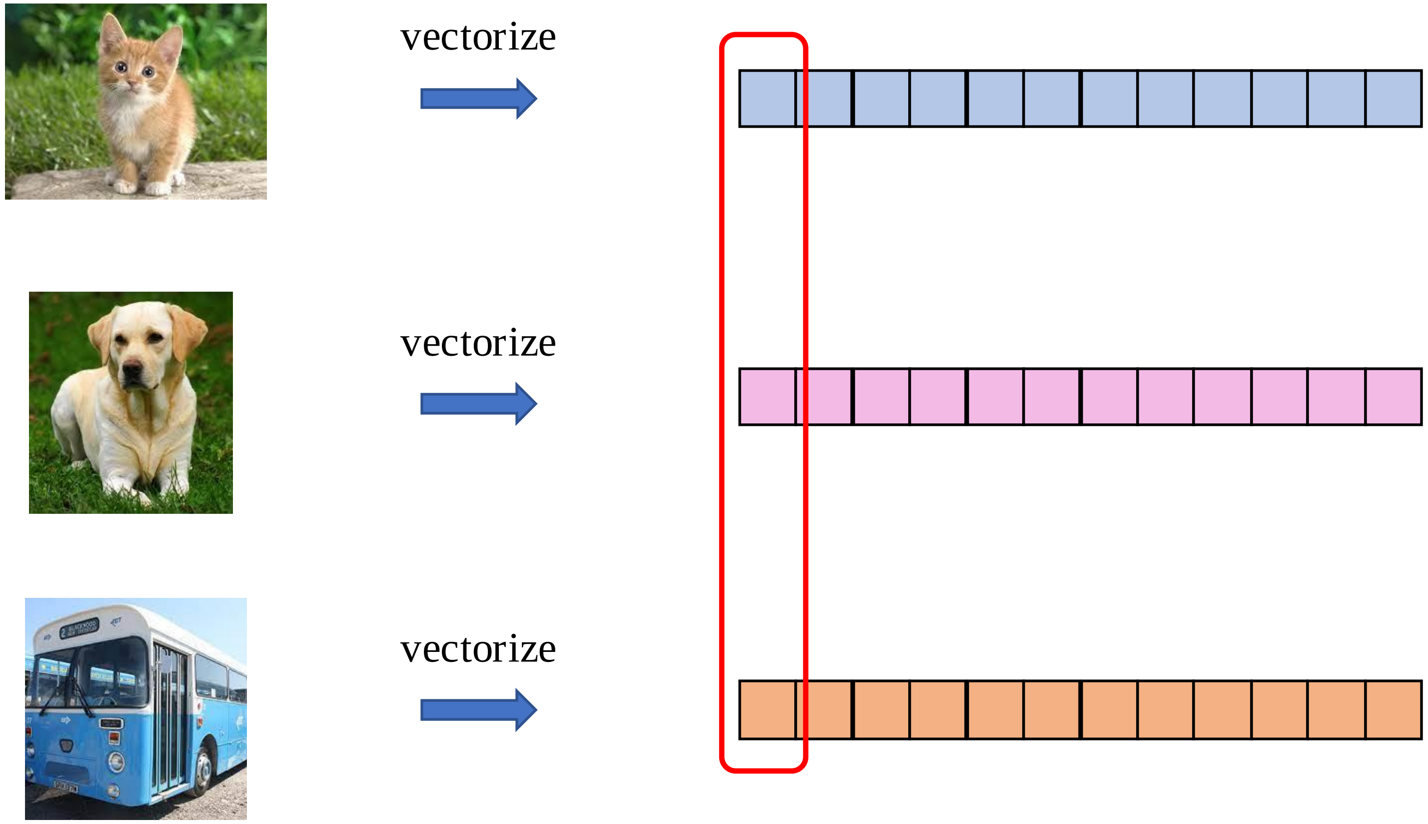

$$\mathbf{m}[j] = \frac{1}{B} \sum_{i=1}^{B} X[i,j]$$

$$\Downarrow$$

$$\mathbf{m} = \frac{1}{B} \sum_{i=1}^{B} X[i,:]$$

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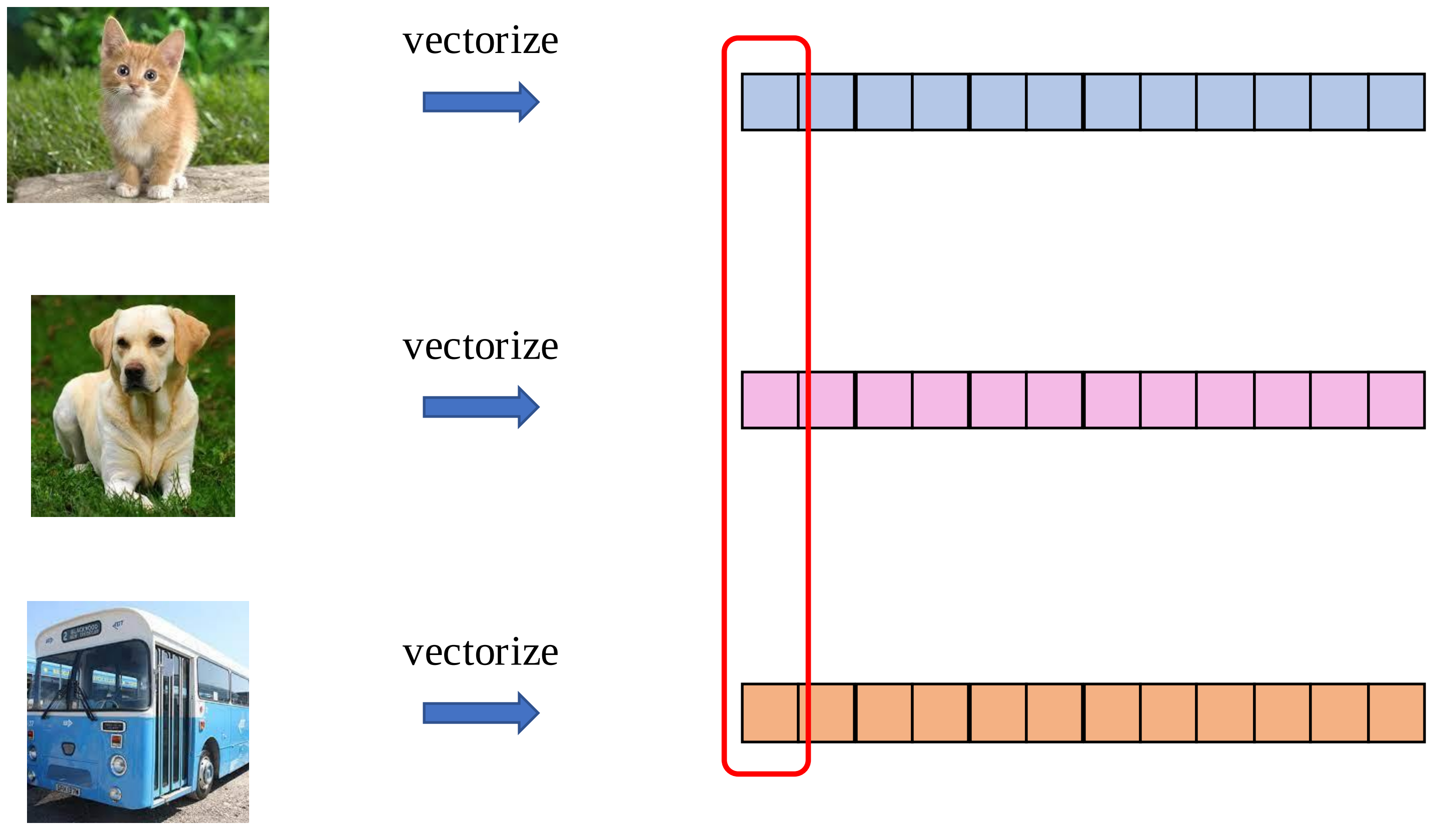

$$\mathbf{m}[j] = \frac{1}{B} \sum_{i=1}^{B} X[i, j]$$

$$\mathbf{v}[j] = \frac{1}{B} \sum_{i=1}^{B} (X[i, j] - \mathbf{m}[j])^2$$

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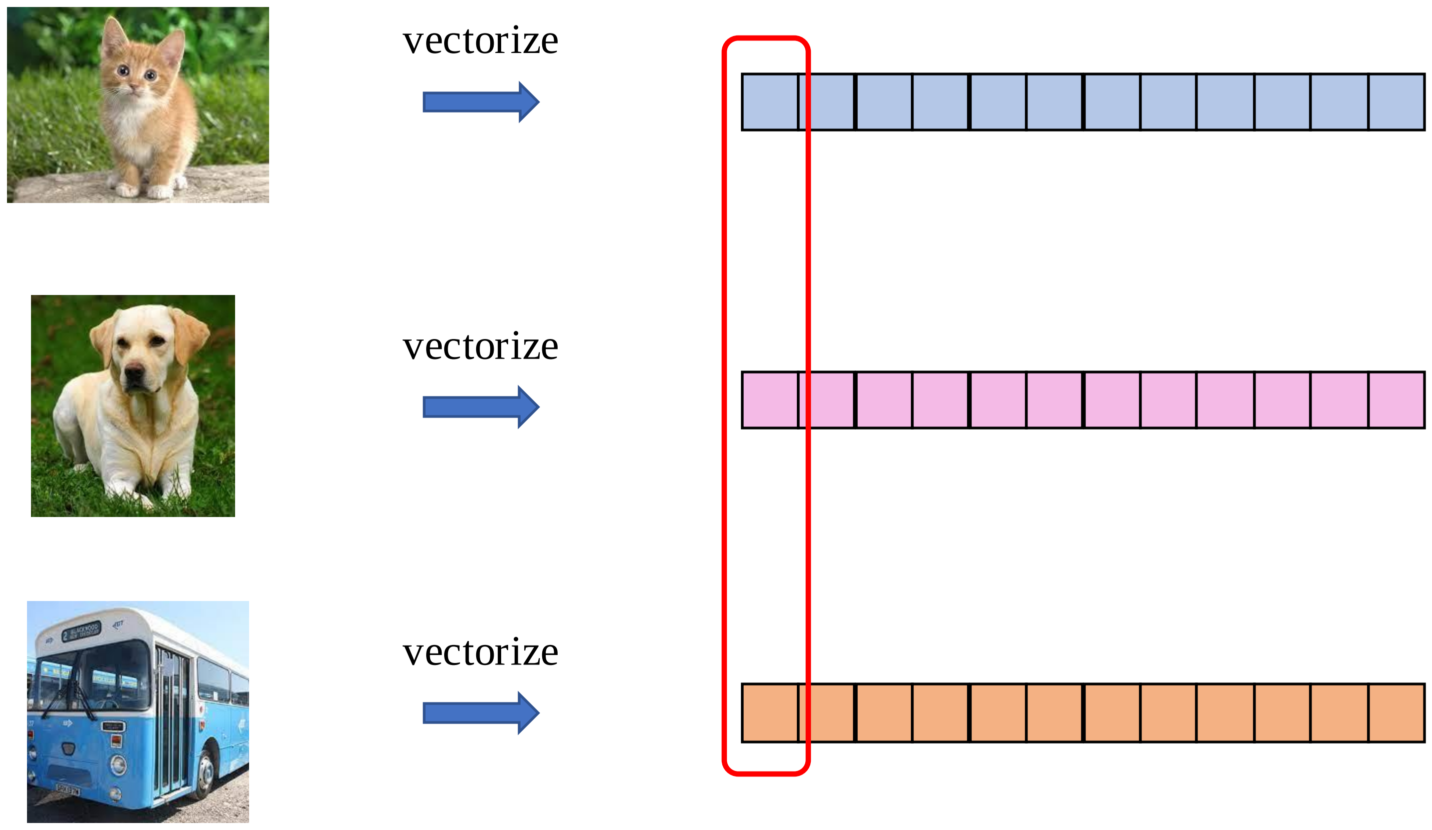

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normalized activations:

$$\hat{X}[i,j] = \boldsymbol{\gamma}[j] \frac{X[i,j] - \mathbf{m}[j]}{\sqrt{\mathbf{v}[j] + \epsilon}} + \boldsymbol{\beta}[j]$$

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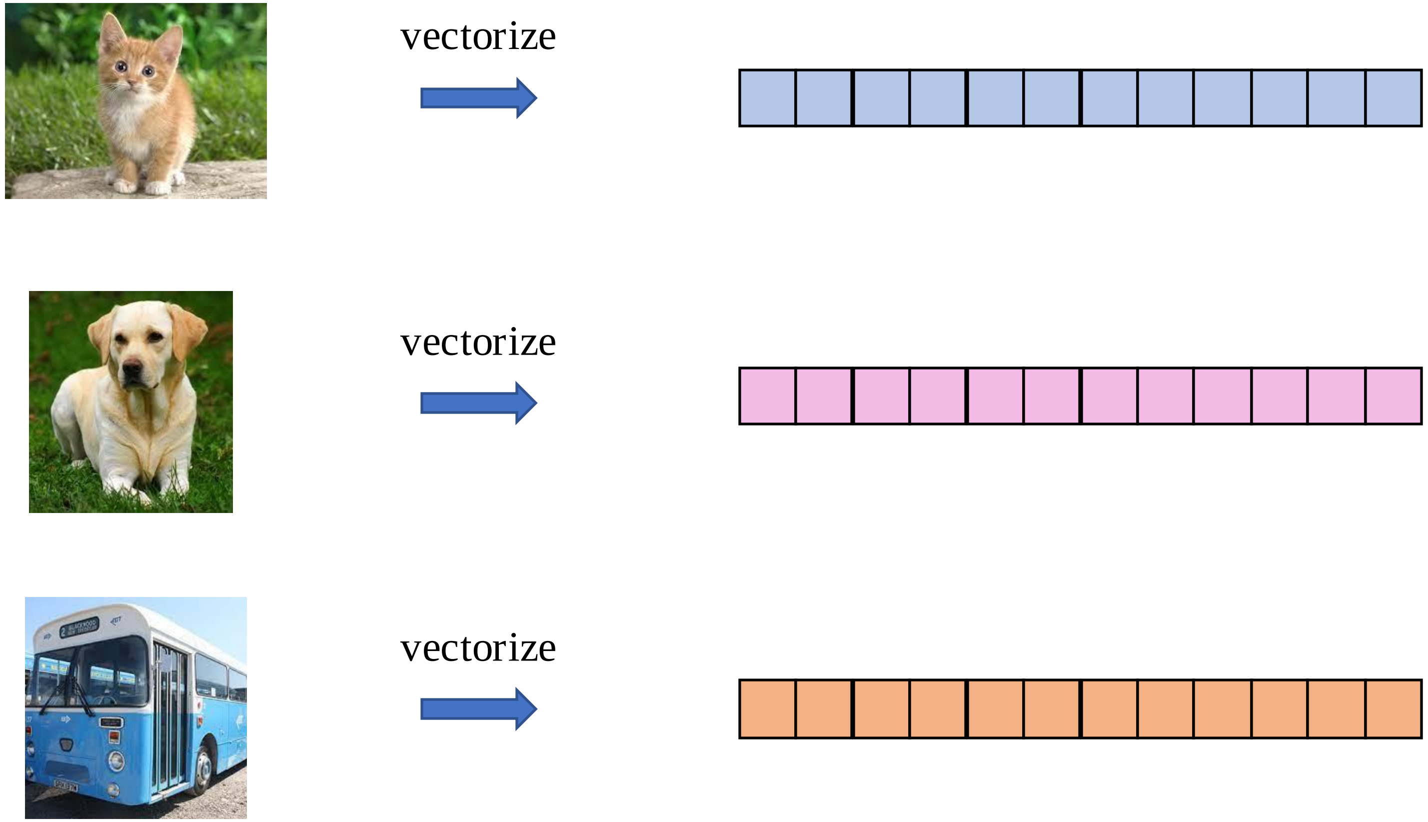

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Learnable Parameters

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In practice, to account for the dynamically changing weights and stochastic data, we use running mean and variance:

$$\mathbf{m}^t[j] = (1-\alpha)\mathbf{m}^{t-1}[j] + \alpha\mathbf{m}[j]$$
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$\alpha \in [0,1]$ is a hyperparameter

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BN can be generalized to convolutional neural networks (CNNs). We will cover that in the future.

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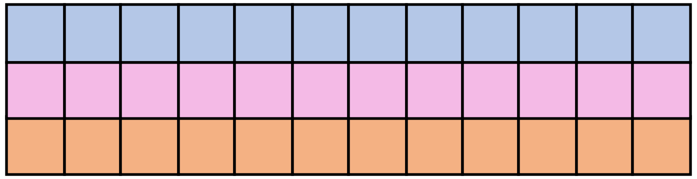
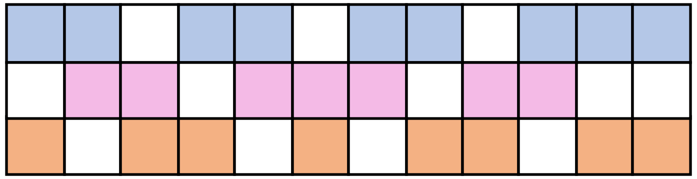


Dropout

Suppose we have a batch of activations $X \in \mathbb{R}^{B \times D}$

We can make them stochastic by randomly turning some units off



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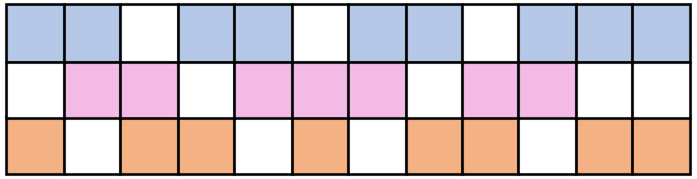

$$M[i, j] \sim \mathrm{Bernoulli}(1 - p)$$

$$\mathbb{P}(M[i, j] = 1) = 1 - p$$
$$\mathbb{P}(M[i, j] = 0) = p$$

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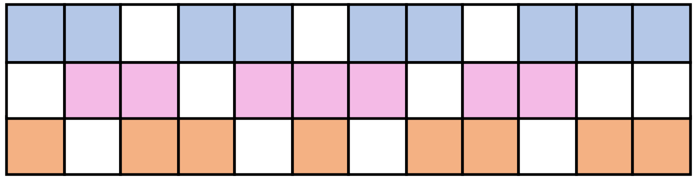
We can make them stochastic by randomly turning some units off

Mathematically, we create a matrix mask $M \in \mathbb{R}^{B \times D}$ as follows



$$M[i,j] \sim \mathrm{Bernoulli}(1-p)$$

$$\mathbb{P}(M[i,j]=1) = 1-p$$
$$\mathbb{P}(M[i,j]=0) = p$$

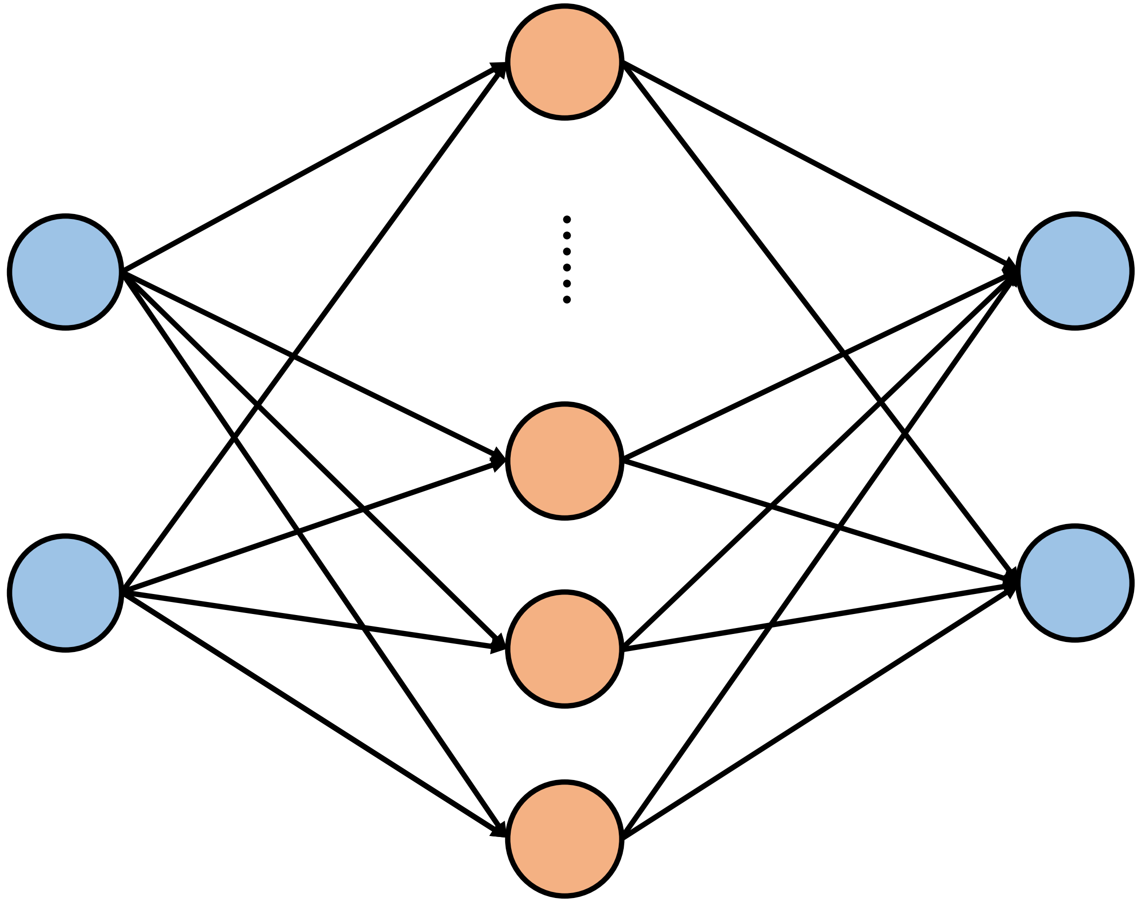
Then we perform element-wise product (a.k.a. Hadamard product)

$$\hat{X} = M \odot X$$

$$\hat{X}[i,j] = M[i,j]X[i,j]$$

Dropout

What does Dropout [8] do to the neural networks?



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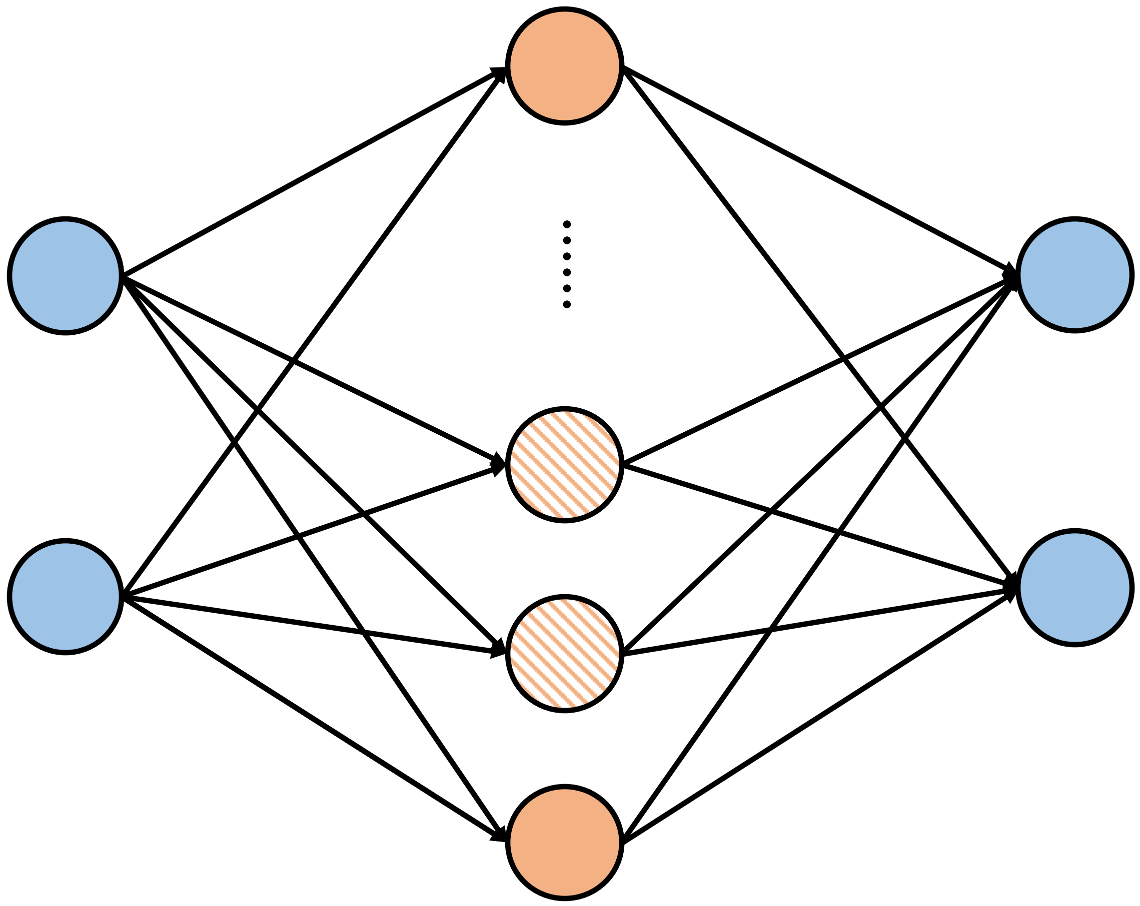

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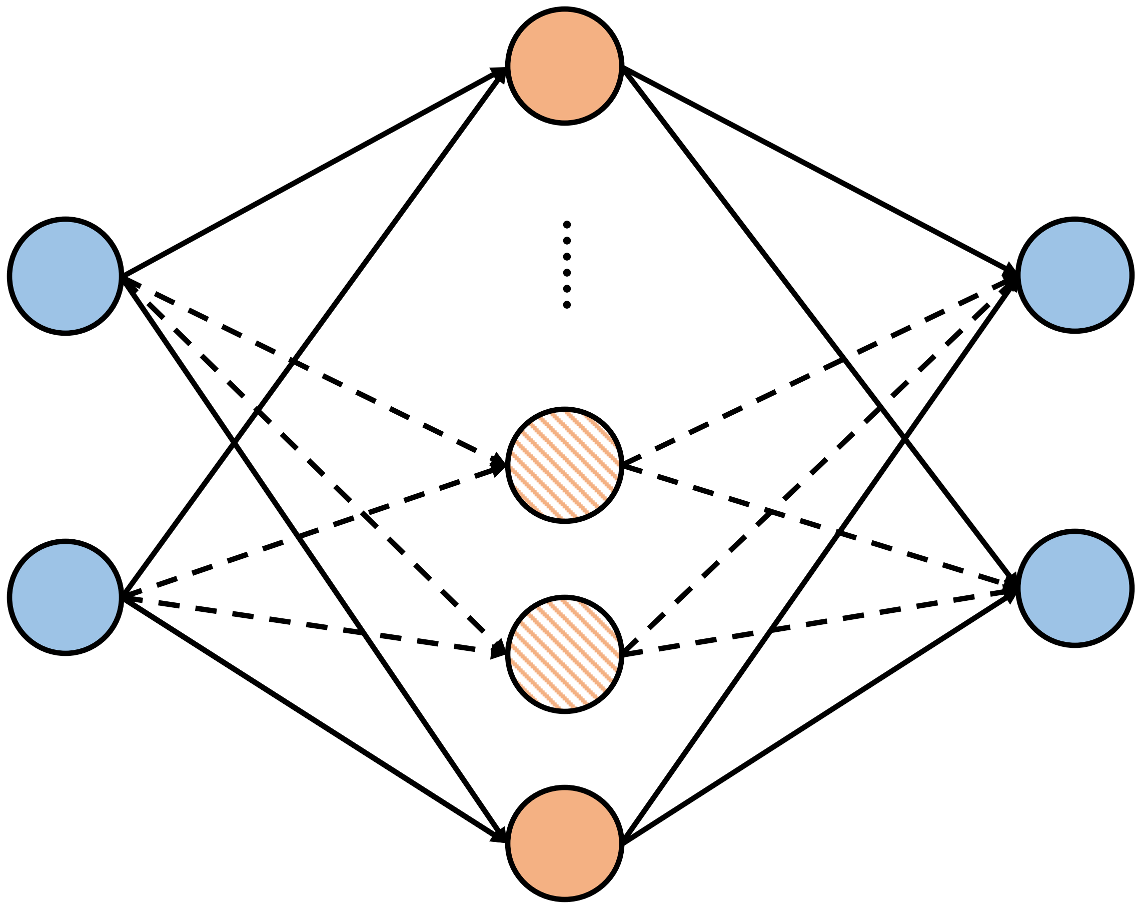

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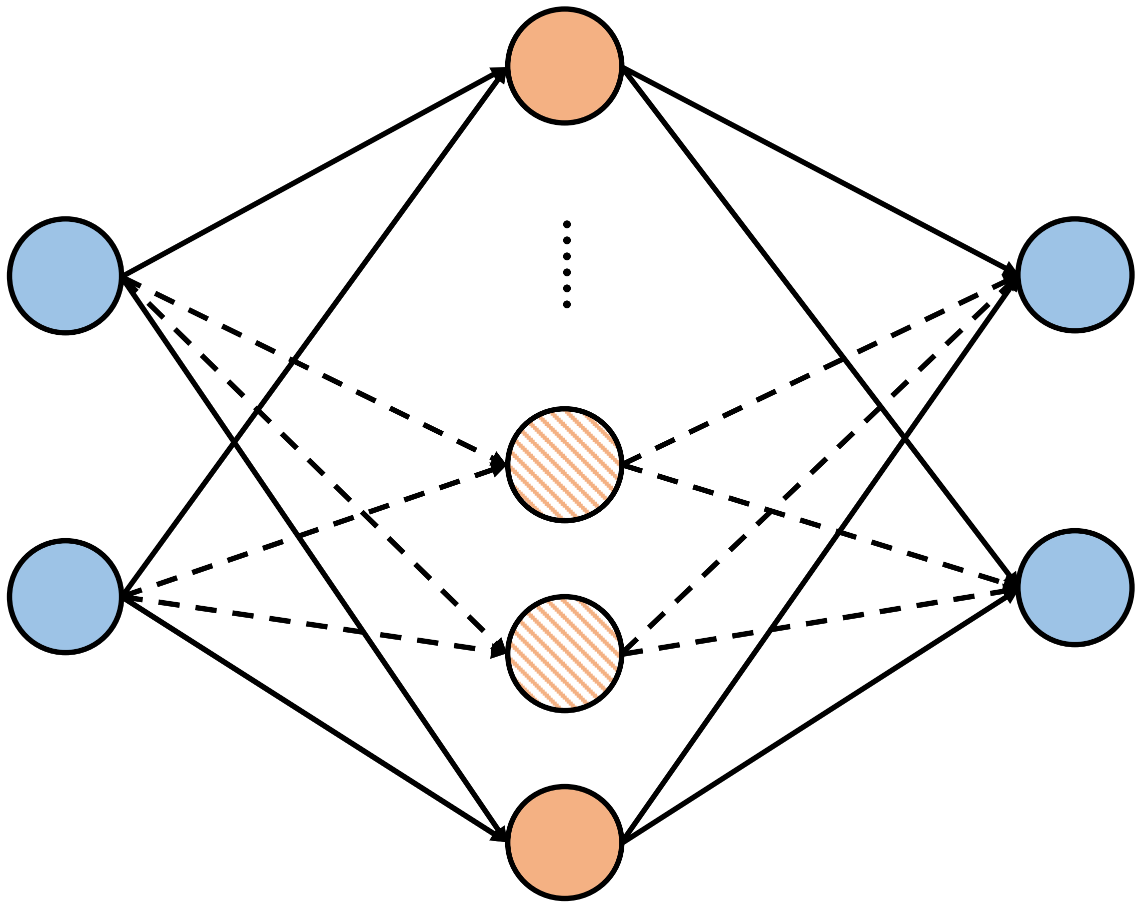
It turns off a random subset of units, thus blocking a random subset of paths!



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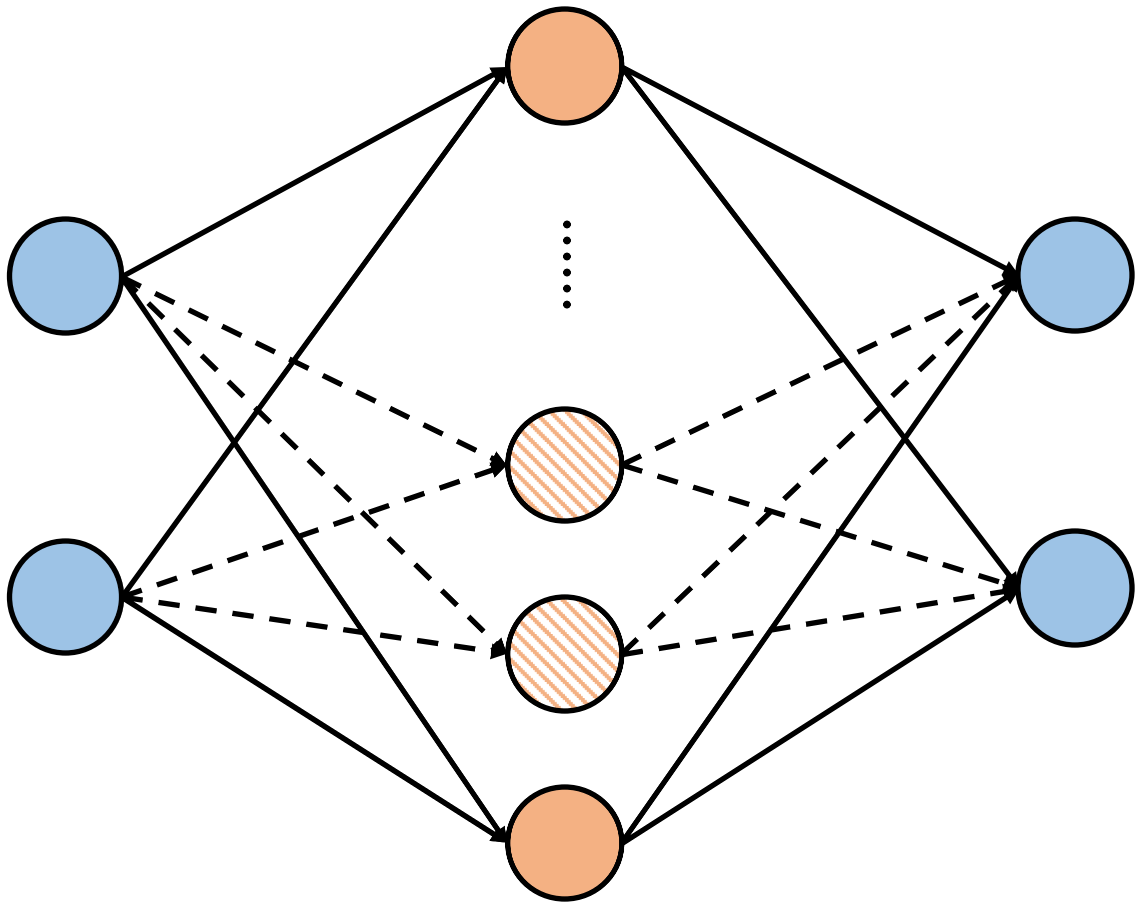
Every sample gets its own sub-network, thus being less likely to overfit.



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We can compute the expectation as

$$\begin{aligned}\mathbb{E}\left[\hat{X}[i,j]\right] &= \mathbb{E}\left[M[i,j]X[i,j]\right] \\ &= \mathbb{E}\left[M[i,j]\right]X[i,j] \\ &= \left(1 \times \mathbb{P}(M[i,j]=1) + 0 \times \mathbb{P}(M[i,j]=0)\right)X[i,j] \\ &= (1-p)X[i,j]\end{aligned}$$

Outline

- Multilayer Perceptron (MLP)
    - Linear Layer
    - Nonlinear Activation
    - Batch Normalization
    - Dropout
    - **Build a Deep MLP**

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Now we learn linear layers, nonlinear activations, batch normalization (BN), dropout, how can we build a deep MLP?

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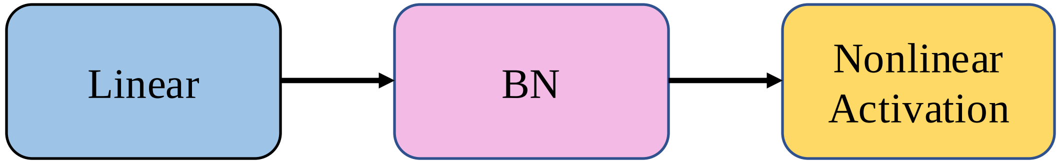

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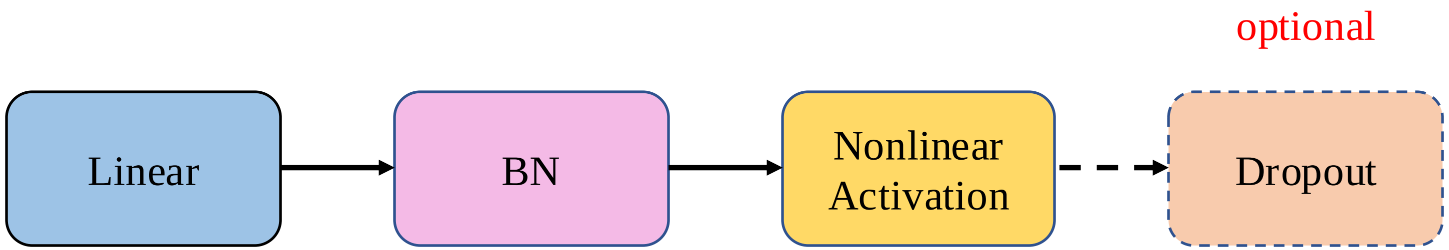

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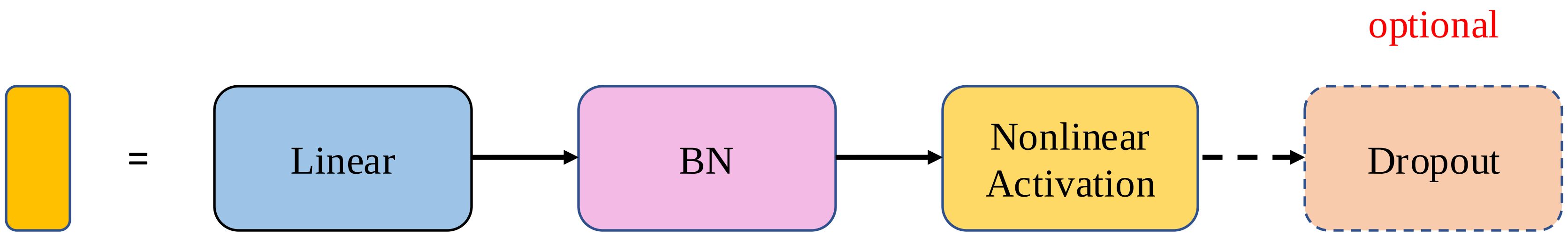

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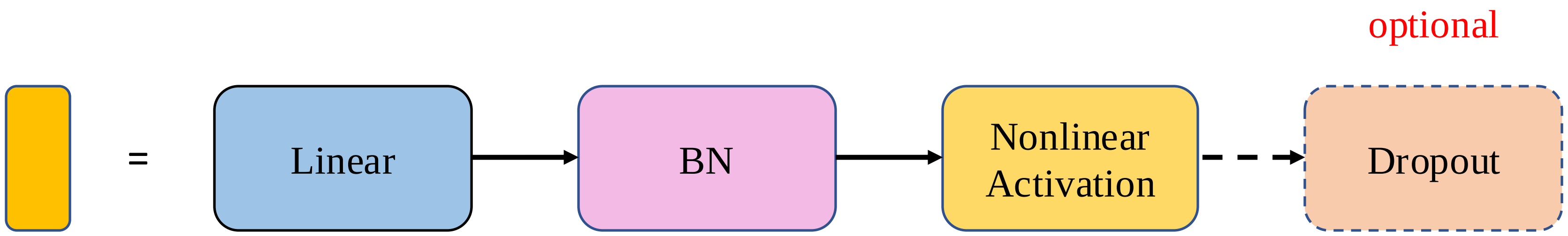

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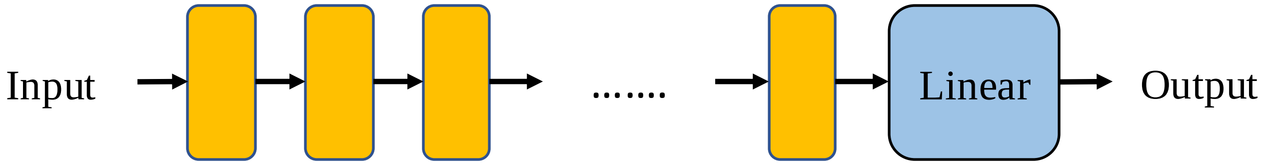

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Questions?